Robustness of the Average Power Ratios in damping estimation: application in SHM of composites beams

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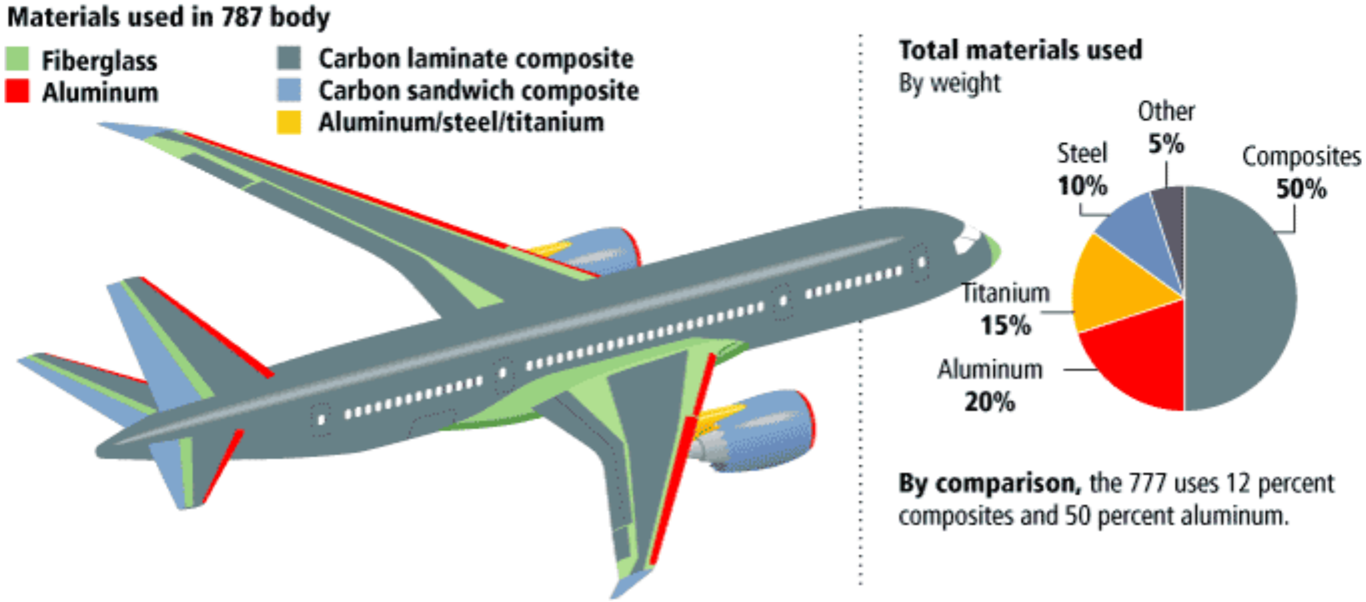
SUPAERO

CST 2010

SHM in Smart Structures (Composites)



Boeing 787

50% of Structure made of Composite

➢ Aim of Using Composites

- ✓ Increase the specific stiffness and strength
- ✓ Reduce the weight

➢ Damage in Composites

- ✓ fabrication stress
- ✓ environmental loadings
- ✓ handling and foreign object impact damage

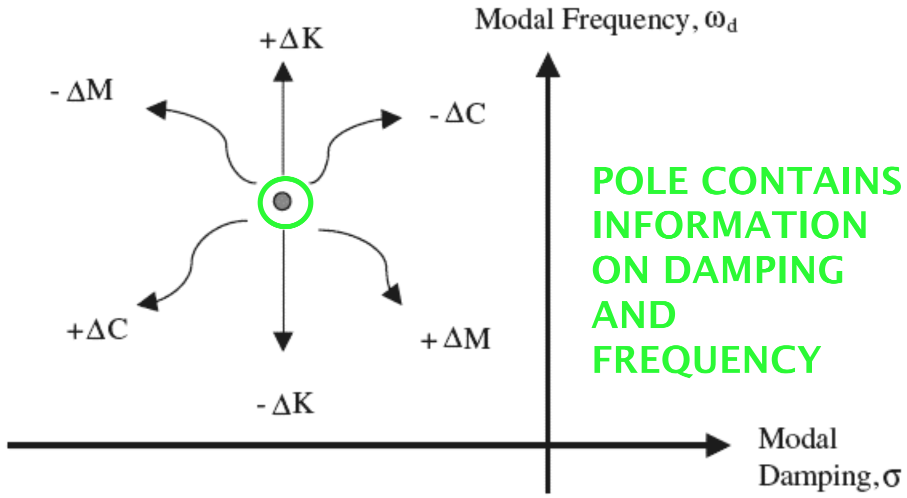
Goals

- Better understanding of performance of composites beams versus LOW ENERGY impact damages (foreign objects, bird strikes, ice …) around BVID (Barely Visible Impact Damage).

- Correlate modal parameters shifts with damage density and level
  - ✓ High Quality vibration tests and drop weight (impact) tests
  - ✓ Results of preliminary works on laminates composites [SHM09]
  - ✓ Model updating and diagnosis tool [SHM10]
  - ✓ Today: Verify new Damping estimator; simple tool for engineer who does not have modal analysis software (LMS, B&K …)

Choice of SHM Detection Techniques

- ➢ Ultrasonic testing
- ➢ Radiography
- ➢ Eddy current testing
- ➢ Liquid penetrant testing
- ➢ Infrared thermography
- ➢ Visual testing (optical)
- ➢ Vibration testing



Purpose of vibration based damage detection

Damage in a structure changes the modal parameters in the following way:

- ✓ Decrease in natural frequency
- ✓ Increase in damping ratio

Outlines

1. Damping estimation using AIPR

2. Vibration Tests & Impact Tests

3. Significance of damage by shifts in modal parameters

4. Conclusions & Future works

Classical bandwidth method

For a linear s.d.o.f. system, the FRF (displacement over force) has the following form:

$$H(\omega) = \frac{1}{-M\omega^2 + \mathrm{j}C\omega + K}, \tag{1}$$

where M, C and K are, respectively, the mass, damping and stiffness constants.

By introducing the undamped natural frequency $\bar{\omega} = \sqrt{K/M}$ and the damping ratio $\zeta = C/(2\sqrt{KM})$, the above equation becomes

$$H(\omega) = \frac{1}{M(-\omega^2 + \mathrm{j}2\zeta\bar{\omega}\omega + \bar{\omega}^2)}. \tag{2}$$

Now let $\omega_{\max}$ be the peak amplitude frequency near the natural frequency

$$\omega_{\max} = \bar{\omega}\sqrt{1 - 2\zeta^2} \tag{4}$$

and $H_{\max} = |H(\omega_{\max})|$ the local maximum amplitude of $H(\omega)$ at $\omega_{\max}$,

$$H_{\max} = \frac{1}{2M\bar{\omega}^2\zeta\sqrt{1-\zeta^2}}. \tag{5}$$

For the sake of simplicity, let the inverse power ratio to be introduced

$$\beta = \beta(\omega) = \frac{1}{\alpha(\omega)} = \frac{H_{\max}^2}{|H(\omega)|^2}. \tag{7}$$

Taking account of Eqs. (2) and (5), one can obtain the following expression of β:

$$\beta = \frac{(\omega^2 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2\omega^2}{4\zeta^2(1-\zeta^2)\bar{\omega}^4}. \tag{8}$$

Classical bandwidth method

By introducing the following dimensionless variable

$$\Omega = \frac{\omega^2}{\bar{\omega}^2} \tag{9}$$

the expression of the inverse power ratio becomes simpler

$$\beta = \frac{(\Omega - 1)^2 + 4\zeta^2\Omega}{4\zeta^2(1-\zeta^2)}. \tag{10}$$

So for a given value of the power ratio, the corresponding frequencies can be determined by solving the following equation of second degree:

$$\Omega^2 + 2(2\zeta^2 - 1)\Omega + 1 - 4\zeta^2(1-\zeta^2)\beta = 0. \tag{11}$$

The discriminant of this equation is

$$\Delta = 16\zeta^2(1-\zeta^2)(\beta - 1) \tag{12}$$

which is linked to the two roots Ω_a and Ω_b of the quadratic equation by

$$\Delta = (\Omega_b - \Omega_a)^2, \tag{13}$$

where Ω_a and Ω_b are defined by Eq. (9):

$$\Omega_a = \frac{\omega_a^2}{\bar{\omega}^2}, \quad \Omega_b = \frac{\omega_b^2}{\bar{\omega}^2}. \tag{14}$$

Now if both β and Δ are known, the damping ratio can be determined by solving Eq. (12) which can be written in the following form:

$$\zeta^4 - \zeta^2 + \frac{\Delta}{16(\beta - 1)} = 0. \tag{15}$$

For $0 < \zeta < \sqrt{2}/2$, the root of the last equation is (see appendix for details)

$$\zeta = \frac{\sqrt{2}}{2}\sqrt{1 - \sqrt{1 - \frac{\Delta}{4(\beta - 1)}}}. \tag{16}$$

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Classical bandwidth method

By taking Eq. (13) into account, one obtains the exact bandwidth formula of damping determination for a s.d.o.f. system

$$\zeta = \frac{\sqrt{2}}{2}\sqrt{1 - \sqrt{1 - \frac{(\Omega_b - \Omega_a)^2}{4(\beta - 1)}}}. \tag{17}$$

This exact formula is to be compared with the classical approximated formula

$$\zeta \approx \frac{\Omega_b - \Omega_a}{4\sqrt{\beta - 1}}. \tag{18}$$

In fact, this approximated formula can be deduced from the exact formula by developing it in series and neglecting high order terms.

Finally, adding the following approximation:

$$\omega_a + \omega_b \approx 2\bar{\omega} \tag{19}$$

leads to the formula

$$\zeta \approx \frac{\omega_b - \omega_a}{2\bar{\omega}\sqrt{\beta - 1}}, \tag{20}$$

where $\bar{\omega}$ can be approximated by $\omega_{\max}$.

In practice, α is taken to be one half, thus $\beta = \beta(\omega_a) = \beta(\omega_b) = 2$, then Eq. (20) becomes the well known half power bandwidth formula

$$\zeta \approx \frac{\omega_b - \omega_a}{2\bar{\omega}}. \tag{21}$$

Classical bandwidth method and average inverse power ratio

Inverse power ratio defined from a FRF

Classical bandwidth method

$$\rho = \rho(\omega_a) = \rho(\omega_b)$$

$$\rho(\omega) = \frac{H_{\max}^2}{|H(\omega)|^2}$$

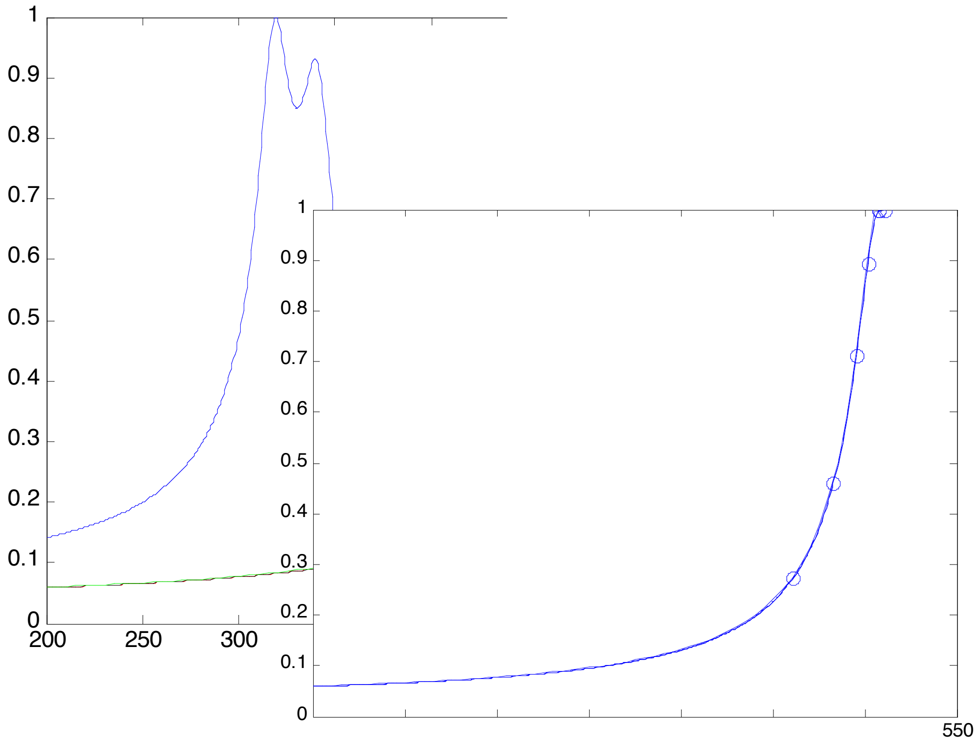
$$\zeta \approx \frac{\omega_b^2 - \omega_a^2}{4\omega_{\max}^2 \sqrt{\rho - 1}}$$

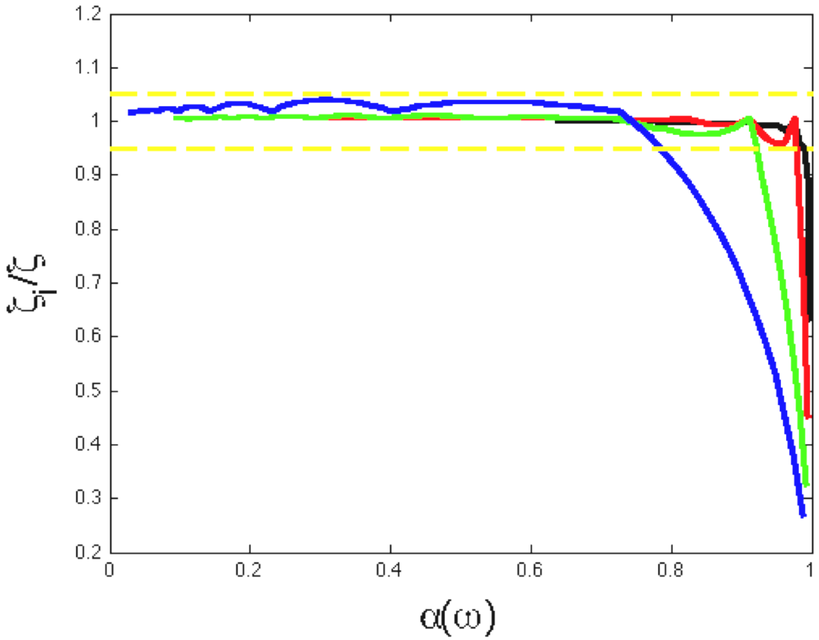
Average inverse power ratio method

$$\omega_{\pm} = \omega_{\max} \pm \delta$$

$$\overline{\rho} = \tfrac{1}{2}\left[\rho(\omega_+) + \rho(\omega_-)\right]$$

$$\zeta \approx \frac{\omega_+^2 - \omega_-^2}{4\omega_{\max}^2 \sqrt{\overline{\rho} - 1}}$$

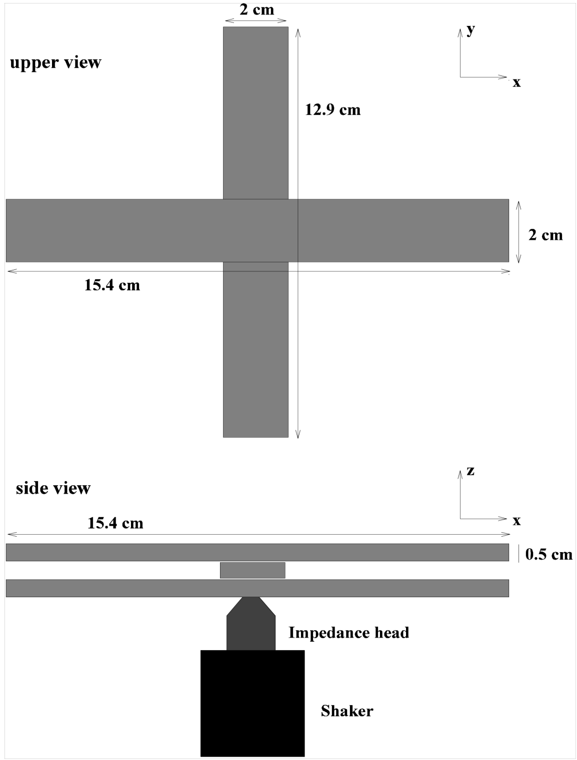


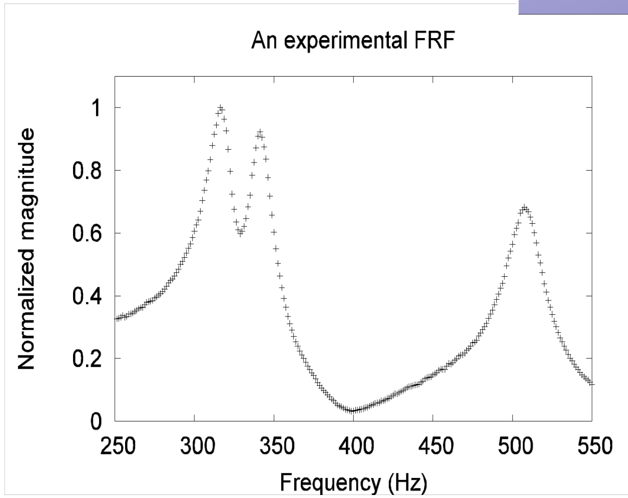


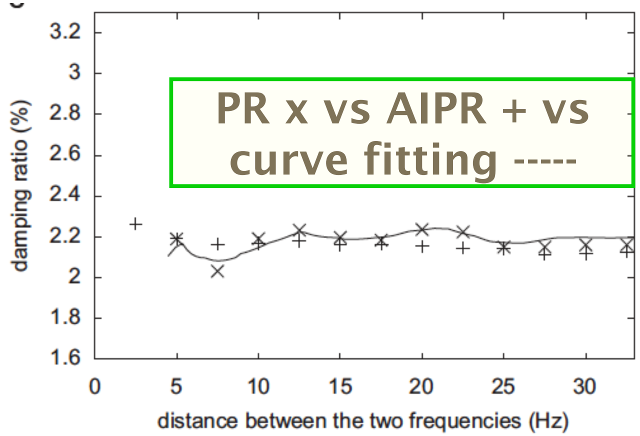

```
function
[etha,rho]=average_power_ratio(H,f,fn,fr)
%H, FRF
%f, Frequency vector
%fn, Resonance
%fr, Sampling frequency

res=0.25
%frequency resolution for interpolation

-> MODE ISOLATION (left side)
-> FIND peak: omega_max

H1 INTERPOLATION of H
H1=H(ima:imb);
f=f(ima:imb);
fi = f(1):res:f(end);
Hi = interp1(f,H1,fi,'linear');
H1=Hi;
f=fi;
df=f(end)-f(1);
n2=round(df/res)

for n=1:n2/2;% every interpolate distance
of omega-max

rho_m=(max(H1)/abs(H1(1+n)))^2;
f_m=f(1+n);

rho_p=(max(H1)/abs(H1(end-n)))^2;
f_p=f(end-n);

rho_a=0.5*(rho_m+rho_p);
rho(n)=rho_a;

etha_a=-(f_m-f_p)/(2*fn*sqrt(rho_a-1));
etha(n)=etha_a;
end
```

A simple model testing case HP Yin MSSP 2009

3 plexiglass beams and the FRF amplitude







PR x vs AIPR + vs curve fitting -----

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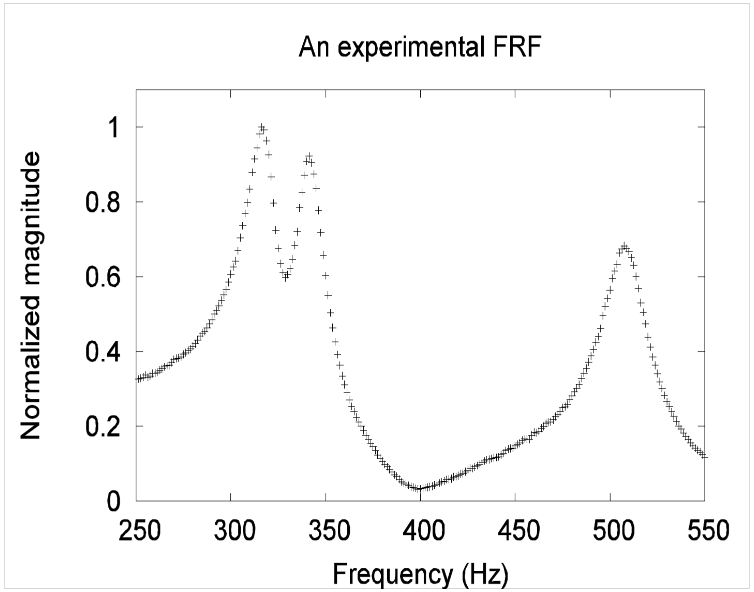
Numerical experiments

Synthetised data

Natural Frequency (Hz)	318.6	341.1	508.5
Frequency Resolution (Hz)	0.25	0.625	1.5
Damping Level (%)	0.1	0.5	2
Signal to Noise Ratio (dB)	20	30	60



Our numerical supervised experiments principally focus on the evaluation of AIPR sensitivity against frequency resolution, SNR (Gaussian White Noise). For comparison we added a well established algorithm RFP

Damping estimation Vs frequency resolution

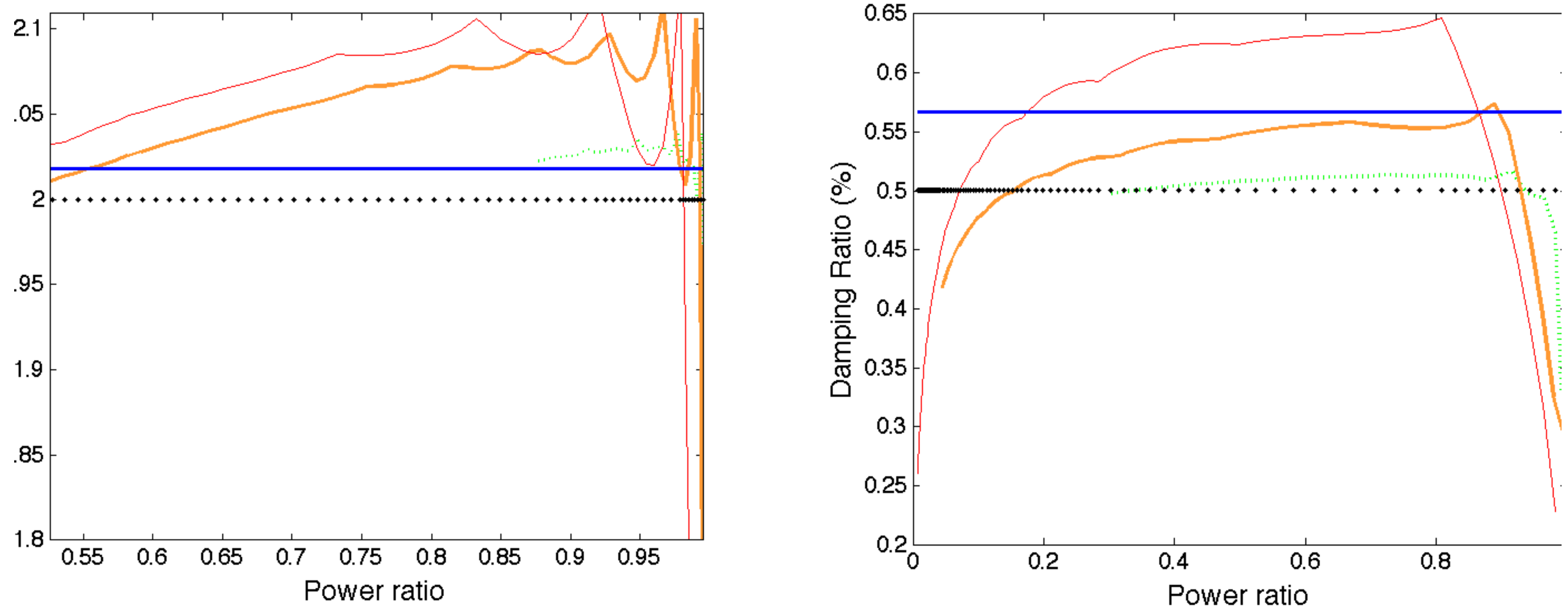


Figure 1: Identified damping ratio on theoretical damping ratio 2% (a) and 0.5% (b) for several value of power ratio for first mode isolated at 318.6 Hz (a) and for third mode (b) at 508.5 Hz. . Thin red line is FR of 1.5 Hz, orange thick line is 0.625 Hz, yellow dotted line is 1 Hz. RFP is represented by blue line very close to theoretical value. AIPR is above 5% of error but always cross the real value at high power ratio.

Damping estimation Vs SNR

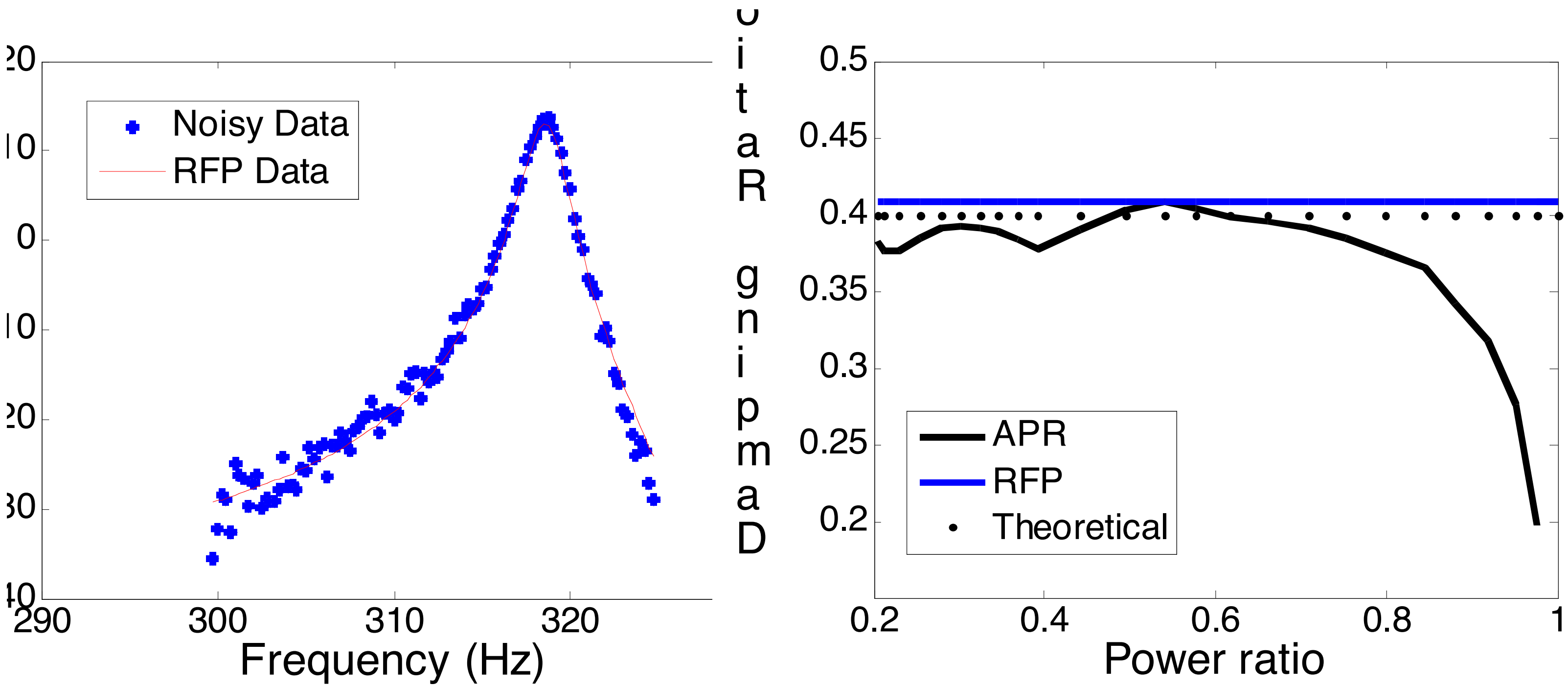
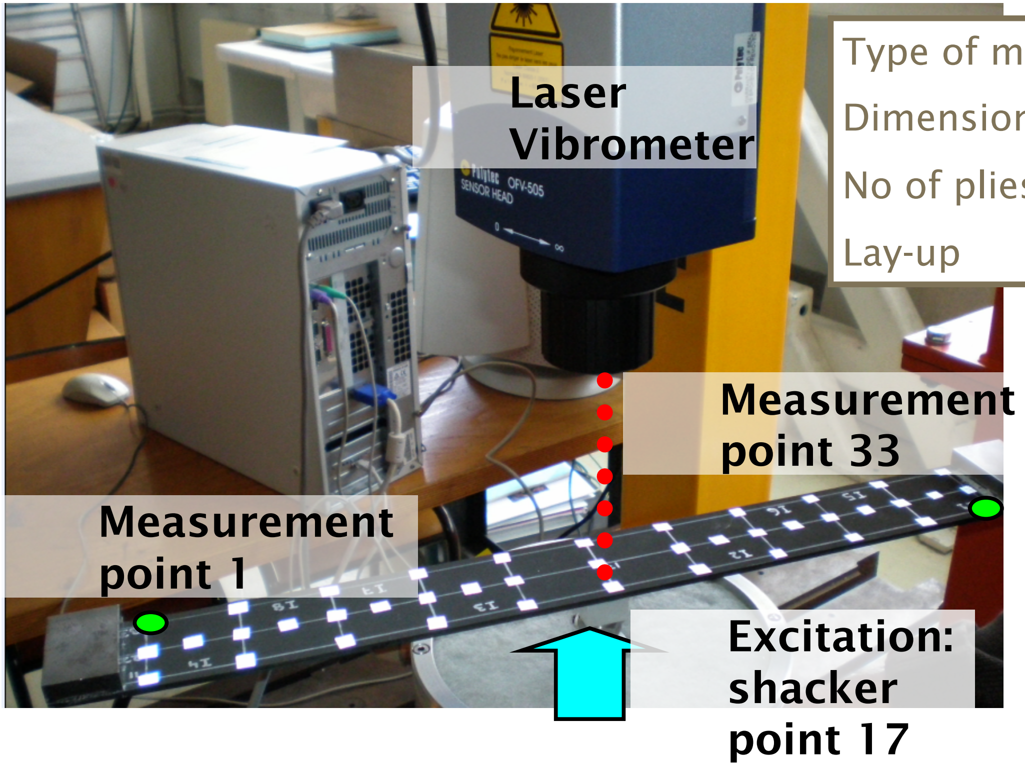


Figure 6: Damping estimation (0.4%) under different realization of a Gaussian noise process (SNR=20dB). Theoretical value is just in the middle of the 2 estimators but the noise has less effect as damping becomes smaller (0.4%)

For low damping (composites material), low power ratio should be preferred (around 0.3), notably dealing with high noise.

Vibration Tests: Testing Methodology

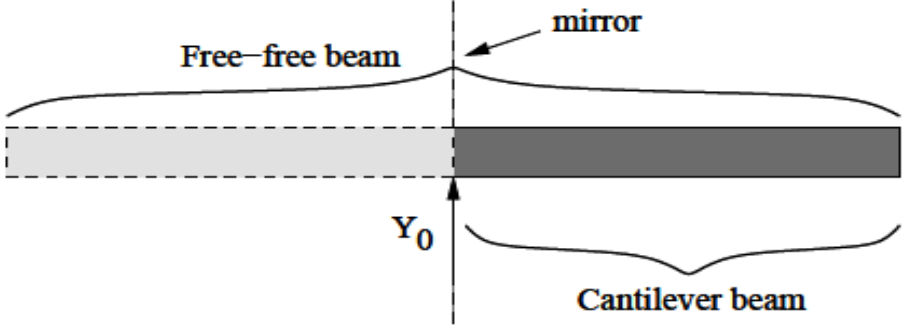


Type of material	T300/914
Dimensions of the beam	480 x 50 x 3 mm
No of plies	24
Lay-up	[0/90/45/-45]3s

➢ **Acquisition Parameters for both BR and SD excitations)**

- ✓ Frequency Resolution = 0.25Hz
- ✓ Excitation level = 1N
- ✓ Same number of measurement points for each beam (33 symmetric points)
- ✓ As the damages are symmetrical, We can compare the sum of FRF

Vibration test based on Oberst Beam method



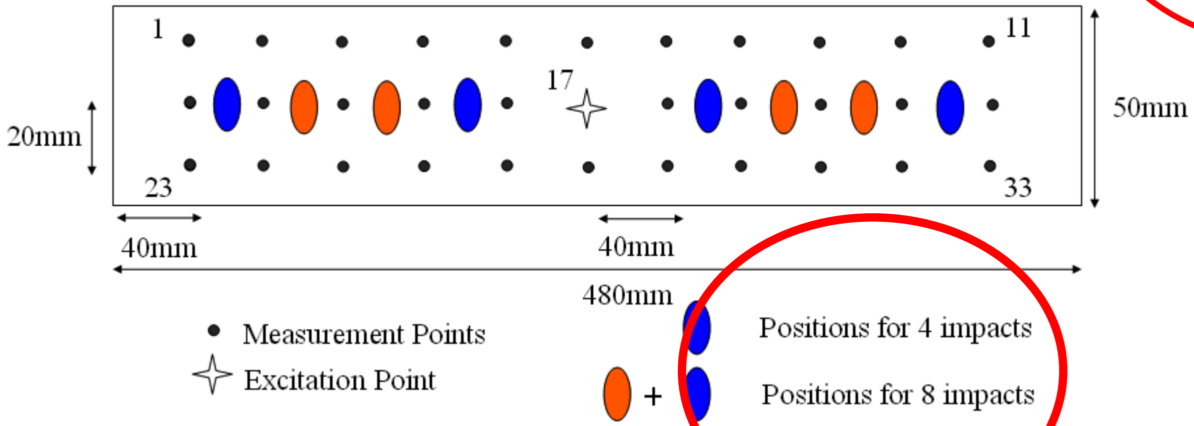
Experiments on 5 identical composites beams

➢ Modal Test Parameters

Type of Excitation	Burst Random
Resolution	0.25 Hz
Frequency bandwidth	0-1600 Hz
Modes of interest	First four bending modes

➢ Impact Test Parameters

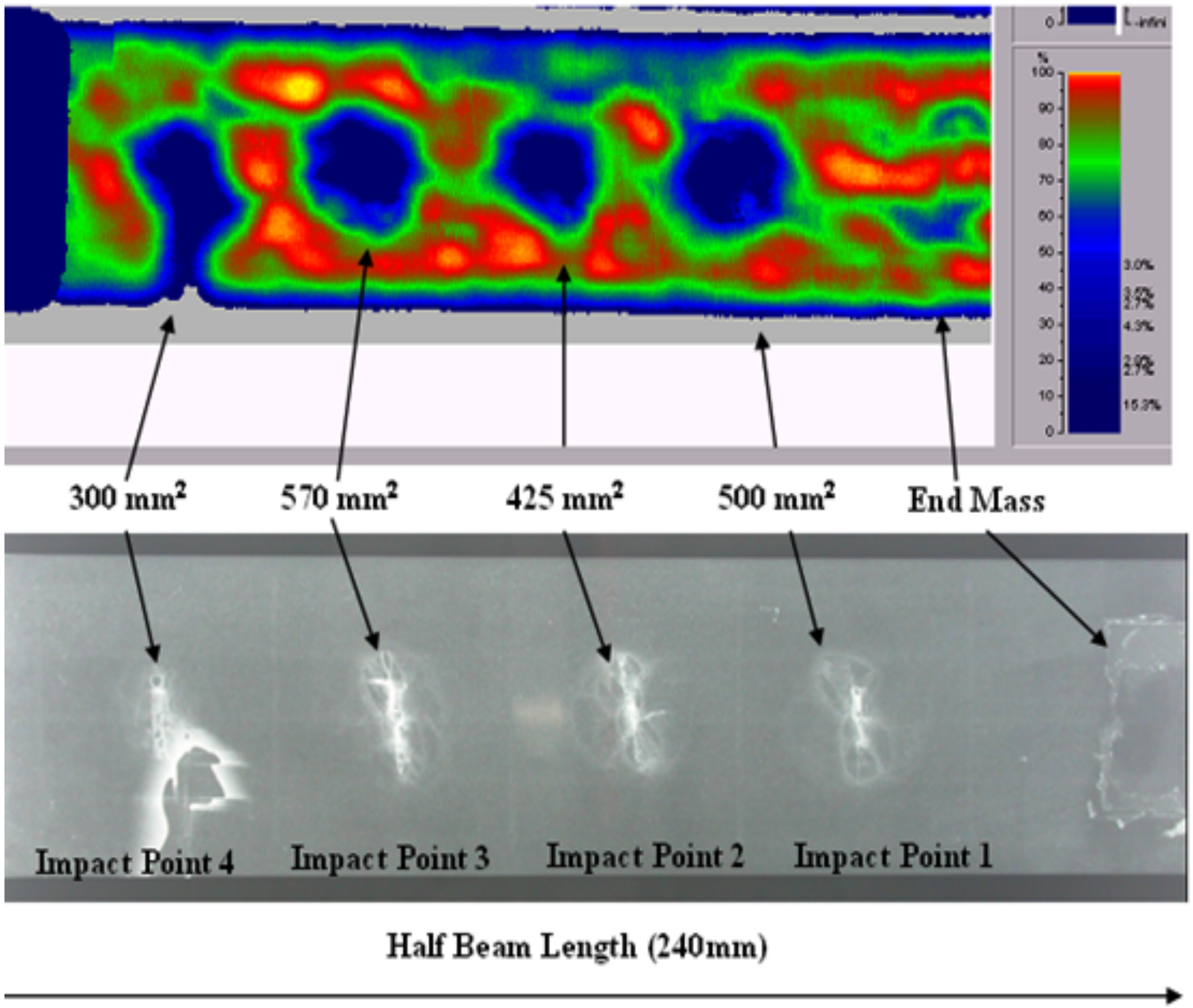
Beam No	Energy of Impact (J)
1	6
2	8
3(BVID)	**10 (0.55mm)**
4	12
5	14



Parameters of the 5x2 Full Factorial DOE

Vibration tests carried out after each of the three states i.e., Undamaged (UD), Damage at 4 points (D1) and at 8 points (D2)

NDT results for impact tests (repetitivity)

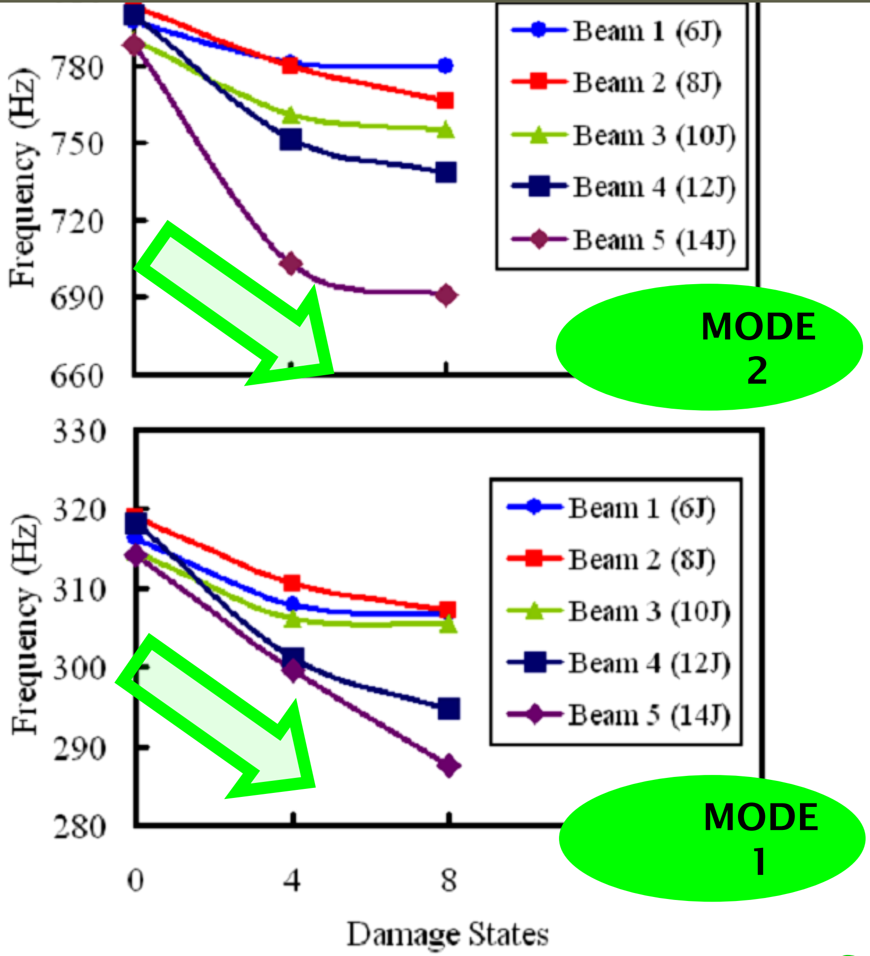


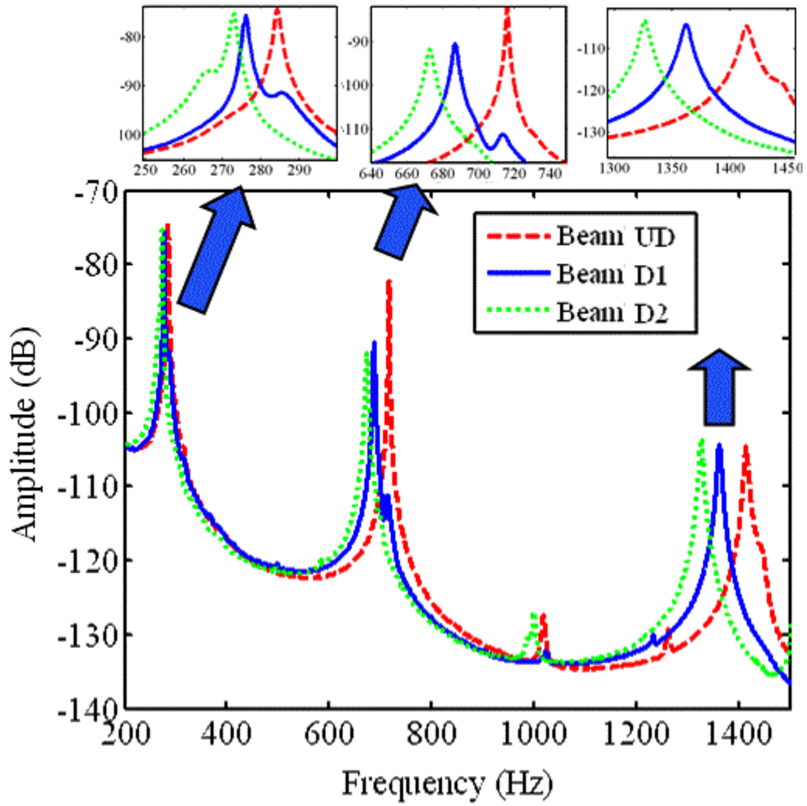
C_SCAN

Damage zones:
local loss of rigidity
(decrease in frequency)
and
increase the surface of friction
(increase in damping)
due to delamination

RADIOSCAN

Shifts in Natural Frequency



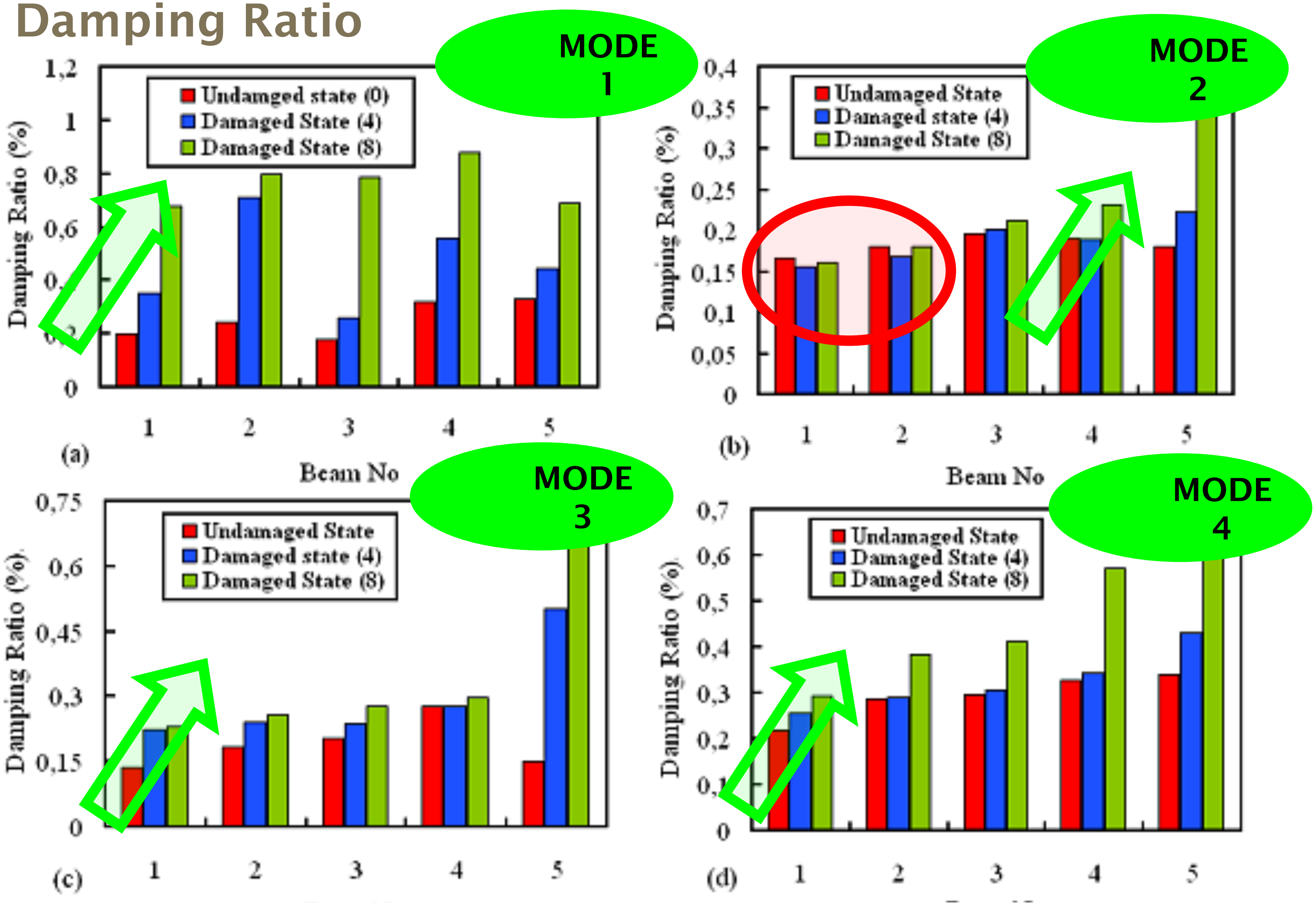


For all modes: Decrease in frequency increases with damage

Shift in frequency is higher for higher modes

Composite Laminates

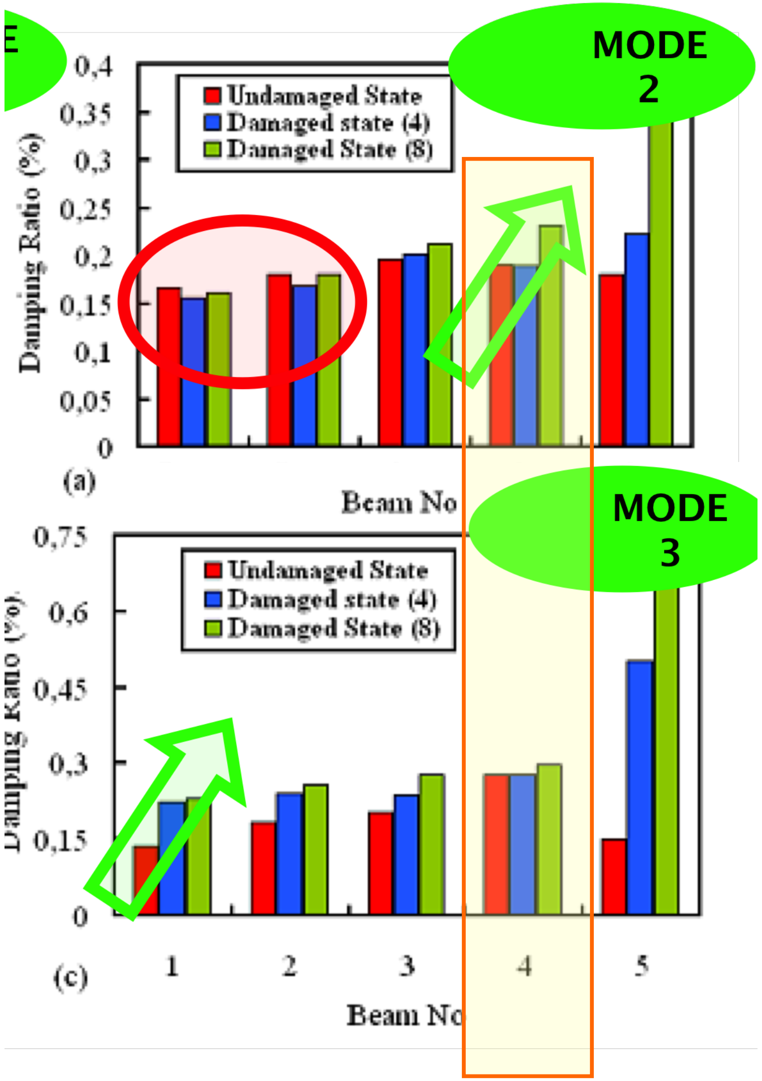
➢ **Shift in Damping Ratio**

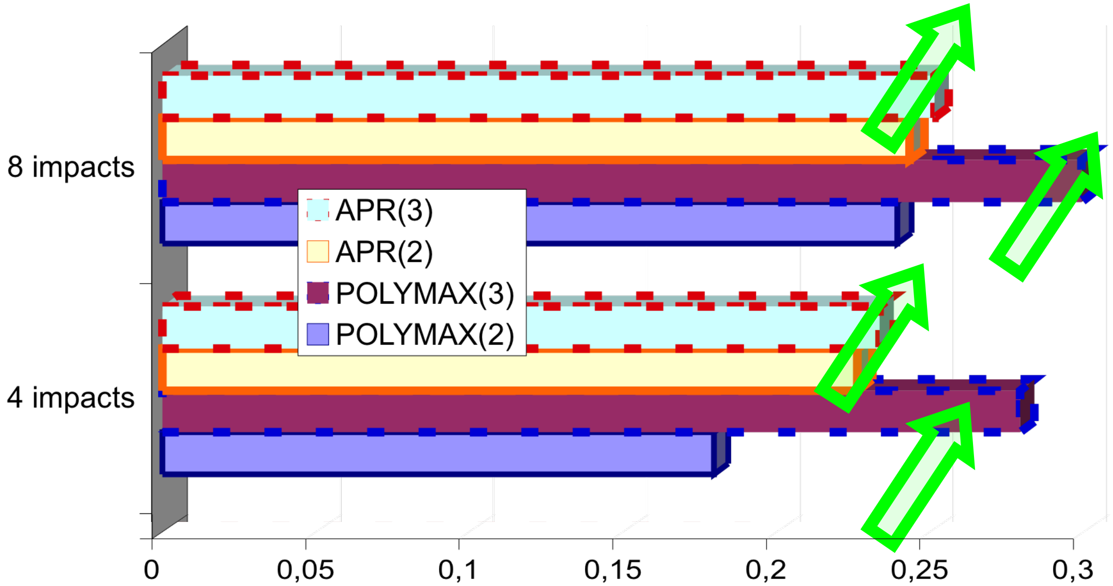


Generally damping increase with damage but sometimes not consistent with damage

Better results with Sine Dwell excitation (non-linearity effects)

Comparison





Generally damping estimation by AIPR is correlated with polymax

But

polymax (use 33FRFs) and

AIPR (5 random FRFs selected with large amplitude of mode)

Conclusions

➢ **Which Modal Parameter is more sensitive to damage ?**

✓ Change in natural frequency = Less than 12%

✓ Change in damping ratio = Up to 200 %

✓ Good correlation between polymax and AIPR

For engineer one advantage of AIPR is the simplicity:
No numerical interpolations

Results shows AIPR is sufficiently accurate respecting several criteria:

- coupling effects are not strong
- frequency resolution is high enough to determine accurately the peak amplitude
- Processing mode by mode, FRF by FRF

✓ Future works: NL behaviour due to impact --> NL modal analysis