

## A Pedagogical Image Processing Tool to Understand Structural Dynamics

*Joseph Morlier*

*ICA ISAE-SUPAERO, 10 avenue Edouard  
Belin, 31055 Toulouse, France*



Institut Supérieur de l'Aéronautique et de l'Espace

# Virtual Measurement

The main goal is to develop/test a pedagogical framework to analyse vibration from video.

Project students demonstrate that, For this experimental rig, modal test with classical contact accelerometers is difficult/long  
Whereas use virtual measurement is easy

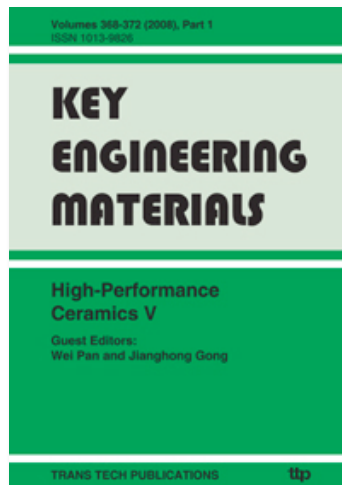
Modeling a fishing rod is difficult (large displacement, NL)  
Comparing different analytical theories (Bernoulli, stepped beam)

- 1 Introduction**
- 2 Optical flow definition**
- 3 Software validation**
- 4 Case study: Dynamic parameters estimation of a flexible beam**

# Virtual Measurement: Previous works

Morlier, Joseph and Salom, Pierre and Bos, Frédéric ( 2007) New image processing tools for structural dynamic monitoring. Key Engineering Materials, vol. 347 . pp. 239-244. ISSN 1013-9826

Morlier, Joseph and Michon, Guilhem ( 2010) Virtual vibration measurement using KLT motion tracking algorithm. Journal of Dynamic Systems Measurement and Control, vol. 132 (n° 1). pp. 011003-011011. ISSN 0022-0434



# OpenCV?

Develop a universal toolbox for research and development in the field of Computer Vision

It is an open source computer vision library developed by Intel.

It focuses mainly towards real-time image processing.

- ▶ Basic structures and operations
- ▶ Image Analysis
- ▶ Structural Analysis
- ▶ Object Recognition
- ▶ **Motion Analysis**
- ▶ **Object Tracking**
- ▶ 3D Reconstruction

-This is the process of locating a moving object in time using a camera

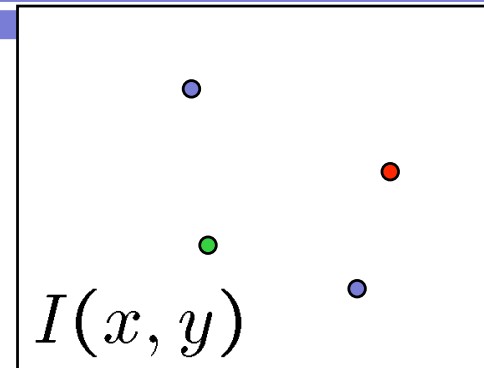
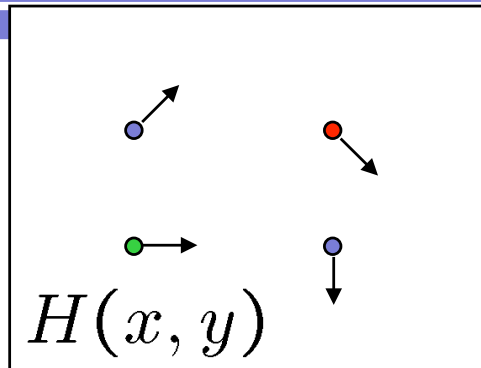
-An algorithm analyzes the video frames and outputs the location of moving targets within the video frame.



## **2. Optical flow definition**

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## How to estimate pixel motion from image H to image I?



Solve pixel correspondence problem

- given a pixel in H, look for **nearby pixels** of the **same color** in I

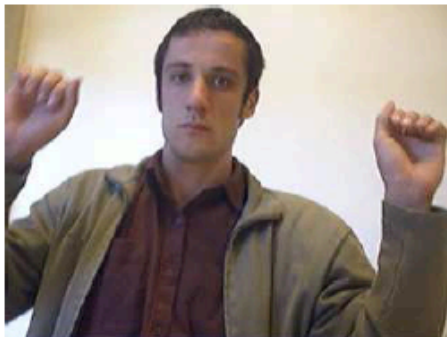
### ► Key assumptions

- **color constancy**: a point in H looks the same in I
  - For grayscale images, this is **brightness constancy**
- **small motion**: points do not move very far

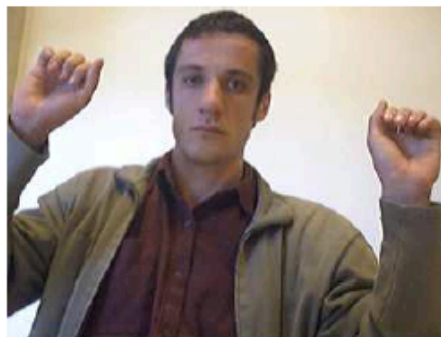
### ► This is called the **optical flow** problem

## Optical Flow

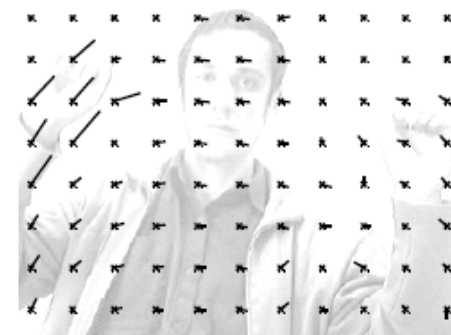
- If a vision-driven-gadget covers a region  $W$  we would like to estimate the 2D displacement  $d=[dx \ dy]$  (optical flow) of this patch  $W$  between two consecutive images that are captured by the camera.
- not only detects the movement but also gives us an estimate of the DIRECTION and the SPEED of the movement.



a) a frame from an image sequence



b) the next frame from the sequence



d) optical flow for some image points  
(lines present displacements)

***KLT: The aim is to find the displacement  $d$  that minimizes the dissimilarity.***

The displacement  $d$  is chosen to minimize the dissimilarity between two feature windows, one in image  $I$  and one in image  $J$ :

$$\varepsilon = \iint_W [J(x + d) - I(x)]^2 w(x) dx$$

where  $w$  is the given feature window,

$x = [x, y]^T$  are coordinates in the image and  $d = [dx, dy]^T$  is the displacement.

The weighting function  $w(x)$  is usually set to the constant 1

**Differentiating**

$$\frac{\partial \varepsilon}{\partial d} = 2 \iint_W [J(x + d) - I(x)] \frac{\partial J(x + d)}{\partial d} w(x) dx = 0$$

**Approximating**

$$J(x + d) \approx J(x) + d_x \frac{\partial}{\partial x} J(x) + d_y \frac{\partial}{\partial y} J(x)$$

$$\frac{\partial \varepsilon}{\partial d} = 2 \iint_W [J(x) - I(x) + g(x)^T d] g(x) w(x) dx = 0$$

Where

$$g(x) = \begin{bmatrix} \frac{\partial}{\partial x} J \\ \frac{\partial}{\partial y} J \end{bmatrix}$$

**Rearranging terms yields a linear  $2 \times 2$  system:**

$$Zd = e$$

$$Z = \iint_W g(x)g(x)^T w(x) dx$$

$$e = \iint_W [I(x) - J(x)] g(x) w(x) dx$$

**Displacement  $d$  is obtained by solving this equation using a Newton-Raphson algorithm**

# processing method for dynamic parameter extraction

## Technical assumptions

- High vertical resolution to obtain sufficient deformation
- Plan of study is perpendicular to the structure (small angular errors)
- Camera stability, image not fuzzy
- Good contrast

## High speed camera to obtain a frequency bandwidth of several hundred of Hz

If we assume global motion with constant velocity  $v_x$  and  $v_y$  (in pixels per standard-speed frame) and spatially band limited image with  $B_x$  and  $B_y$  as the horizontal and vertical spatial bandwidths (in cycles per pixel),

the minimum temporal sampling frequency  $f_s$  (in cycles per speed frame) to avoid **motion aliasing** is given by:

$$f_s = 2B_t = 2B_x v_x + 2B_y v_y$$



lkdemoresonance2.cpp

```
    IplImage* eig = cvCreateImage( cvGetSize(grey), 32, 1 );
    IplImage* temp = cvCreateImage( cvGetSize(grey), 32, 1 );
    double quality = 0.01;
    double min_distance = 10;

    count = MAX_COUNT;
    cvGoodFeaturesToTrack( grey, eig, temp, points[1], &count,
                          quality, min_distance, 0, 3, 0, 0.04 );
    cvFindCornerSubPix( grey, points[1], count,
                      cvSize(win_size, win_size), cvSize(-1, -1),
                      cvTermCriteria(CV_TERMCRIT_ITER|CV_TERMCRIT_EPS, 20, 0.03));
    cvReleaseImage( &eig );
    cvReleaseImage( &temp );

    add_remove_pt = 40;
}
else if( count > 0 )
{
    cvCalcOpticalFlowPyrLK( prev_grey, grey, prev_pyramid, pyramid,
                          points[0], points[1], count, cvSize(win_size, win_size), 3, status, 0,
                          cvTermCriteria(CV_TERMCRIT_ITER|CV_TERMCRIT_EPS, 20, 0.03), flags );
    flags |= CV_LKFLOW_PYR_A_READY;
    for( i = k = 0; i < count; i++ )
    {
        if( add_remove_pt )
        {
            double dx = pt.x - points[1][i].x;
            double dy = pt.y - points[1][i].y;

            if( dx*dx + dy*dy <= 25 )
            {
                add_remove_pt = 0;
                continue;
            }
        }

        if( !status[i] )
            continue;

        points[1][k++] = points[1][i];
        cvCircle( image, cvPointFrom32f(points[1][i]), 3, CV_RGB(0,200,0), -1, 4, 0);
    }
}
```

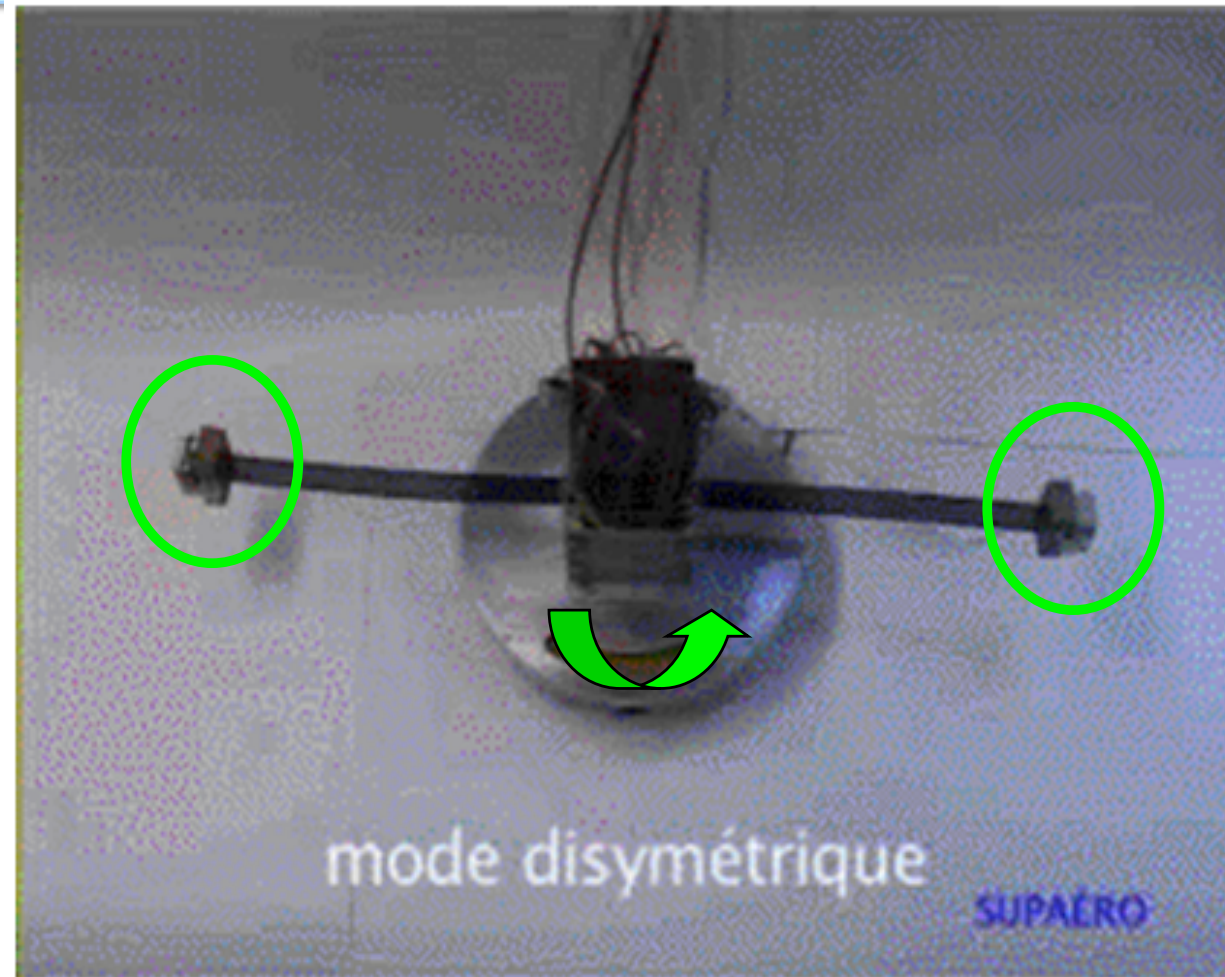
# 3. Software validation

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## Software Validation, normal camera

### Satellite Model, Flexible structure

key points  
-to choose





# Helicopter Blade, Flexible structure

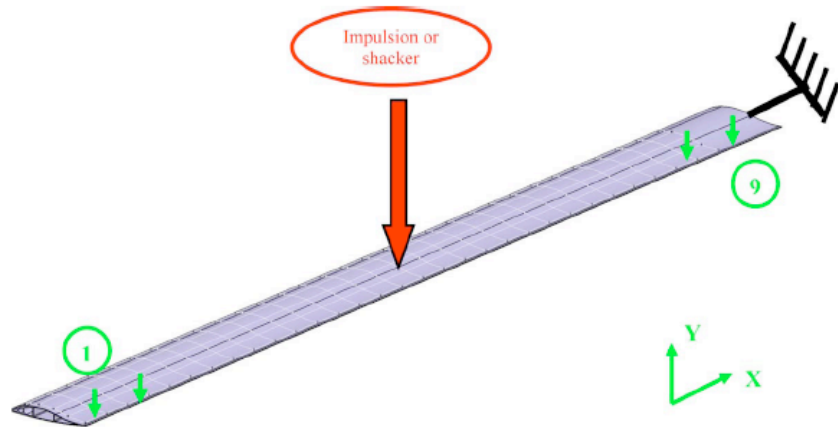


Fig. 6 Helicopter blade example: KLT trackers are used to follow nine targets in bending (Y displacement). The targets are numbered from 1 to 9, and the blade is excited at its center.

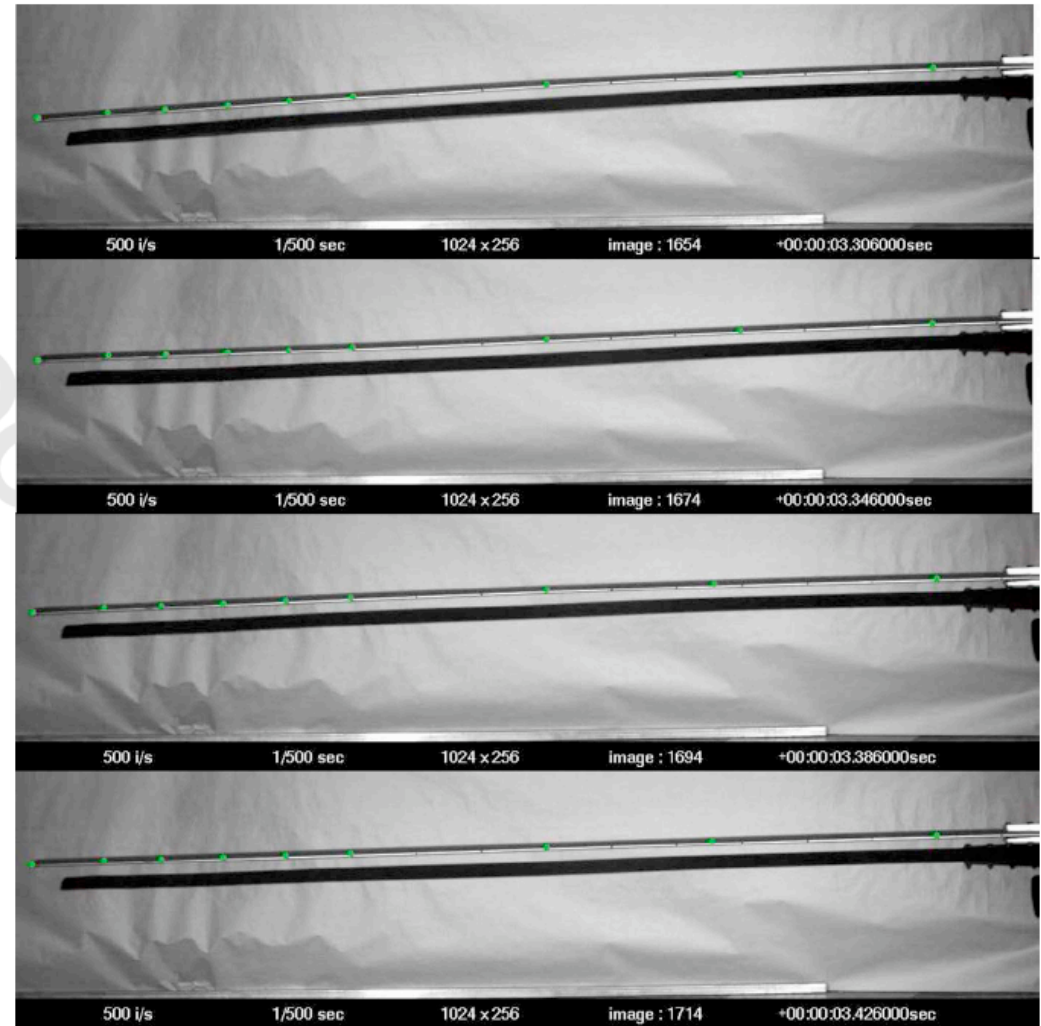


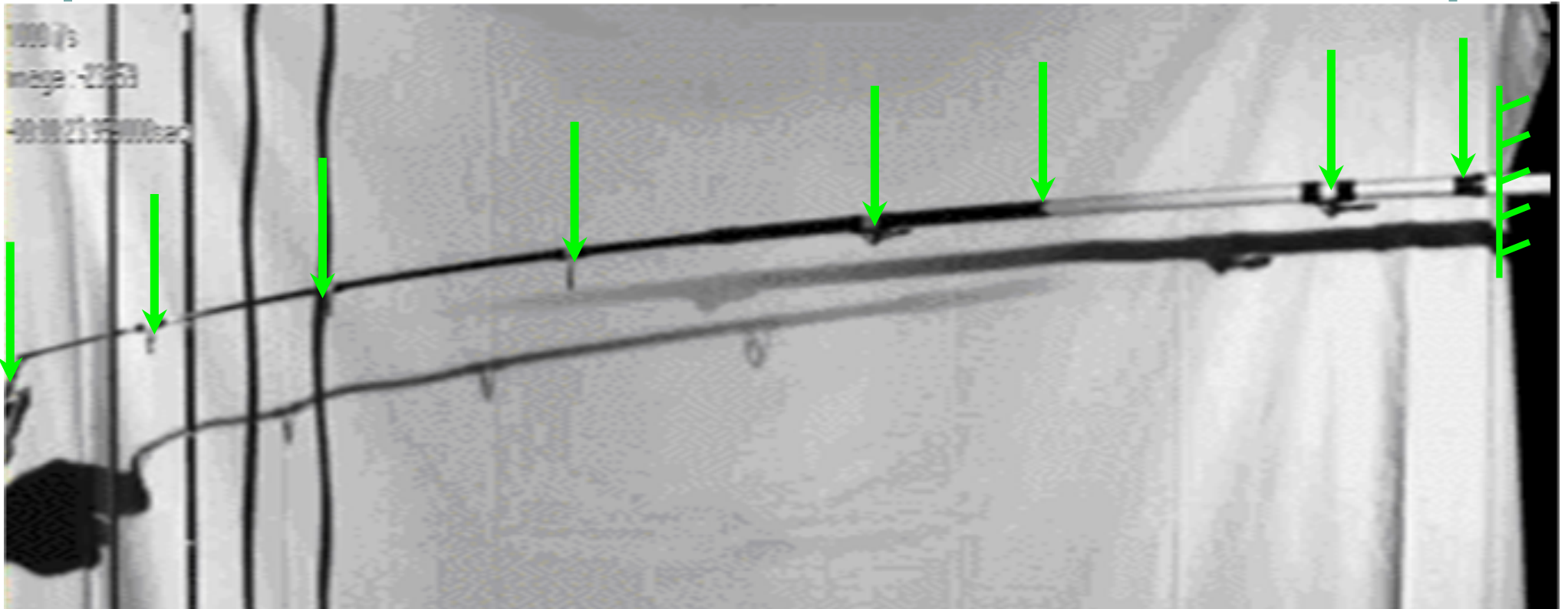
Fig. 10 Half-period is visualized from several successive frames. Virtual sensors are visualized with a small green dot; displacement is measured in the Y direction (green dots).

## **4. Case study**

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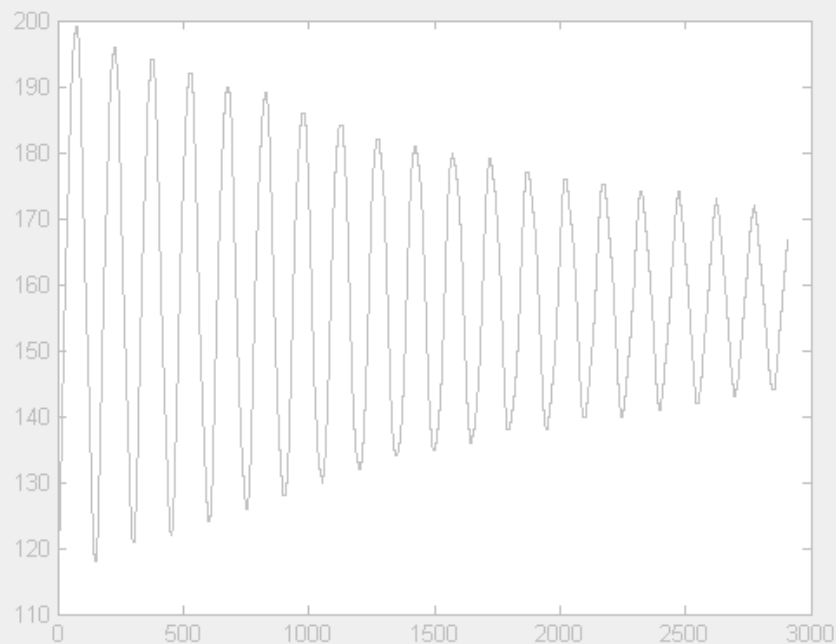
## Pedagogical practice using High Speed Camera

### Fishing rod Test rig (blocked in XY plane)

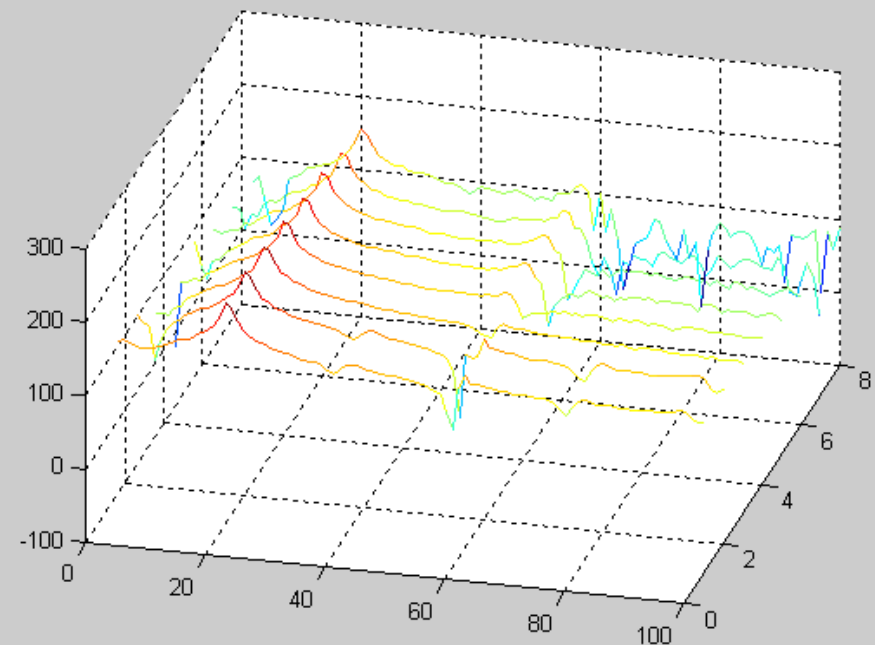


# Analysis of software results

## (High Speed Camera) Fishing rod Free Vibration response



Displacement at the free edge  
(pixels vs frame)



Amplitude of the FRF  
(dB vs Frequency Vs points under study)

## Smoothed temporal data: moving average

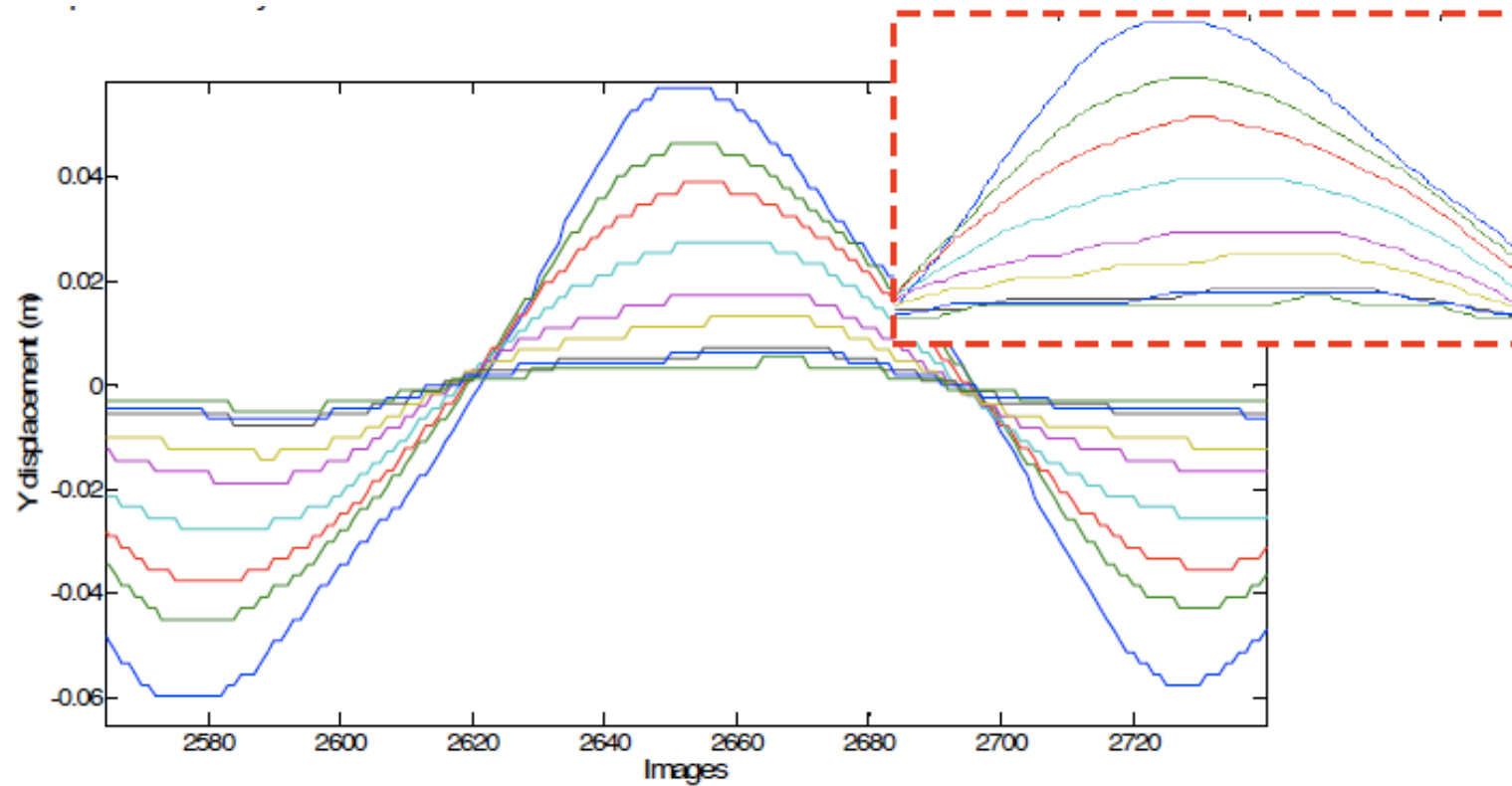
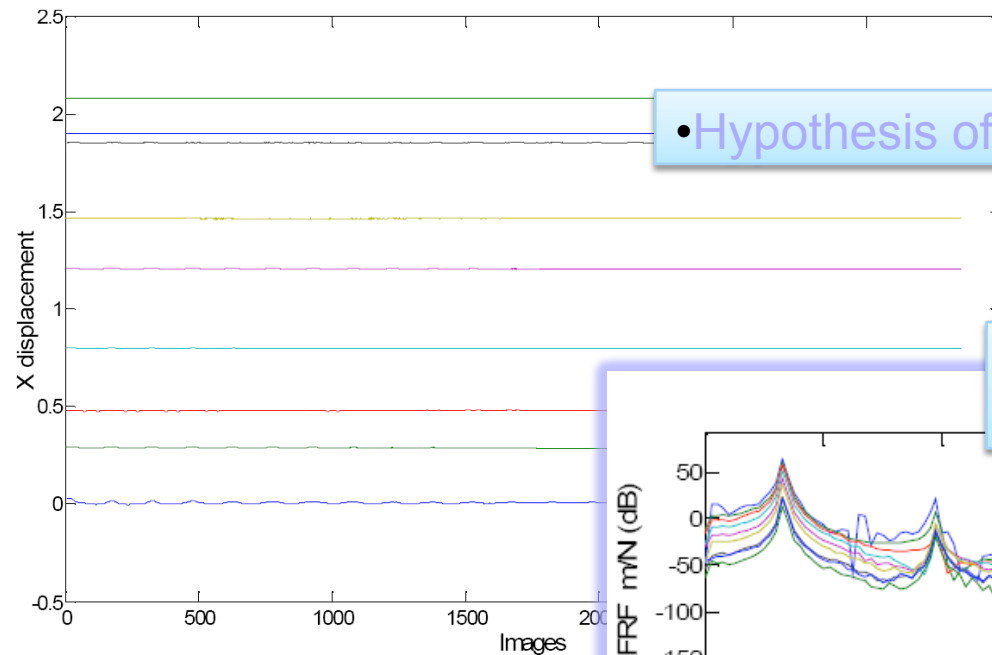


Figure 4: Effect of the moving average function on the temporal signal. This pre-processing aims at obtaining smooth FRFs.

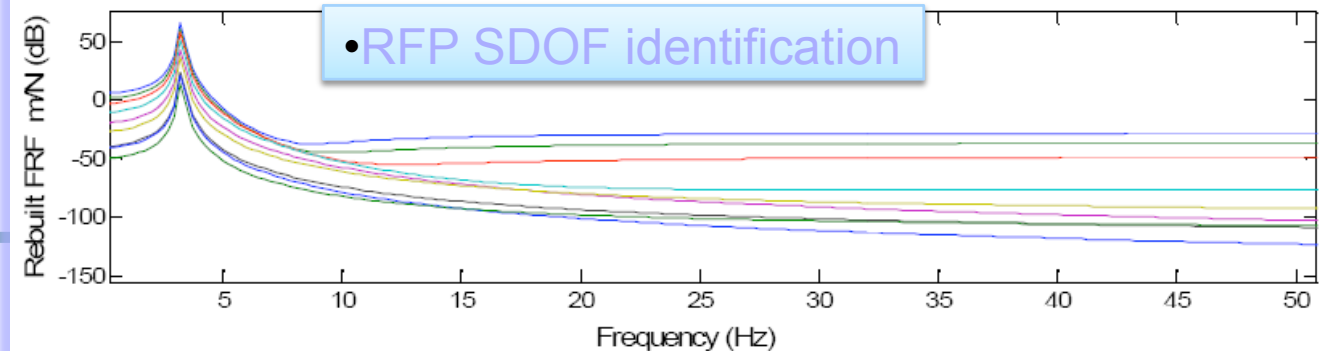
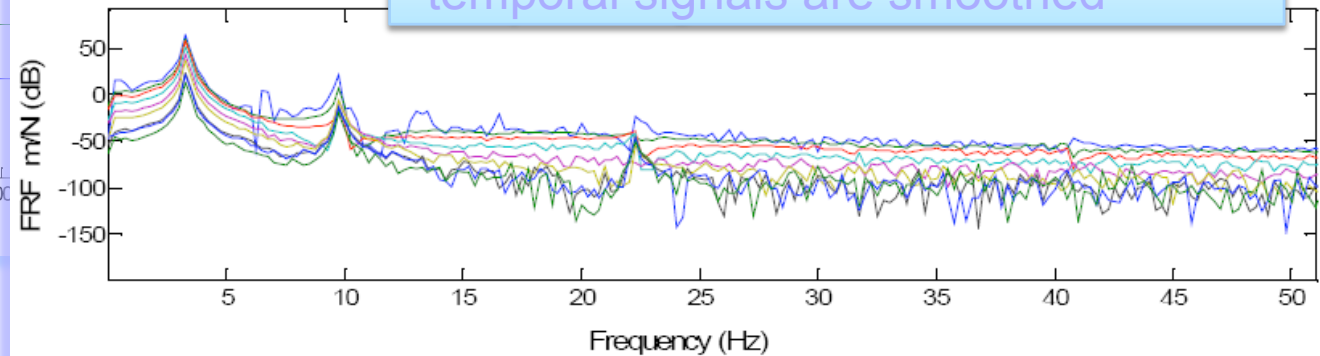
# From video motion estimation to dynamic monitoring



- Hypothesis of small linear displacement

To enhance the spectral resolution :

- temporal signals are smoothed



- RFP SDOF identification

## Results of the motion tracking monitoring

| $E(f)$     | $\sigma(f)$ | $E(\xi)$ | $\sigma(\xi)$ |
|------------|-------------|----------|---------------|
| 3.32 (Hz)  | 6E-4        | 0.93 (%) | 3E-4          |
| 9.78 (Hz)  | 8E-2        | 0.96 (%) | 3E-3          |
| 21.69 (Hz) | 3E-2        | 0.73 (%) | 3E-3          |

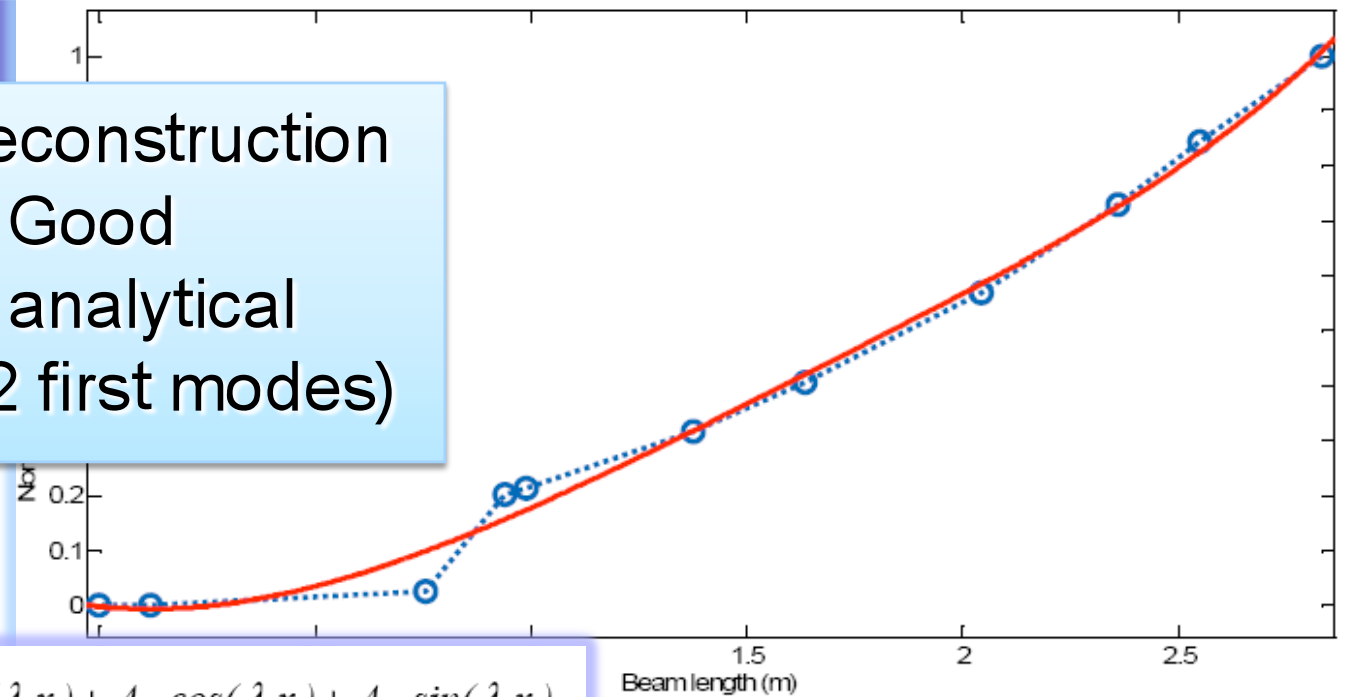
High Correlation with previous accelerometers data

| Frequency ratio  | Tapered theory | Experimental | Error |
|--|----------------|--------------|-------|
| $\frac{\omega_2}{\omega_1} = \left(\frac{\beta_2}{\beta_1}\right)^2$ | 2.61           | 2.945        | 12%   |
| $\frac{\omega_3}{\omega_2} = \left(\frac{\beta_3}{\beta_2}\right)^2$ | 1.81           | 2.21         | 22%   |

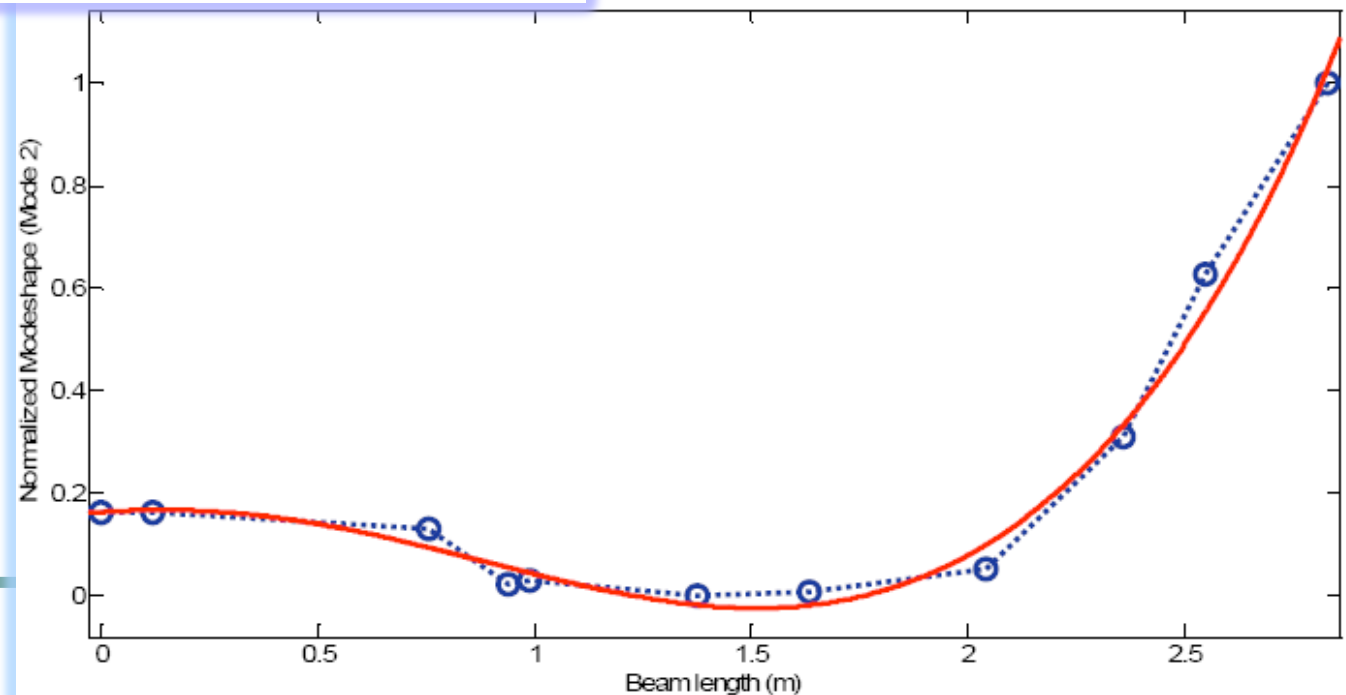
Good correlation with theory

H. Wang and W.J. Worley, Tables of natural frequencies and nodes for transverse vibration of tapered beams, NASA CR443, 1966.

Mode shapes reconstruction from FRF data: Good regression with analytical mode shapes (2 first modes)

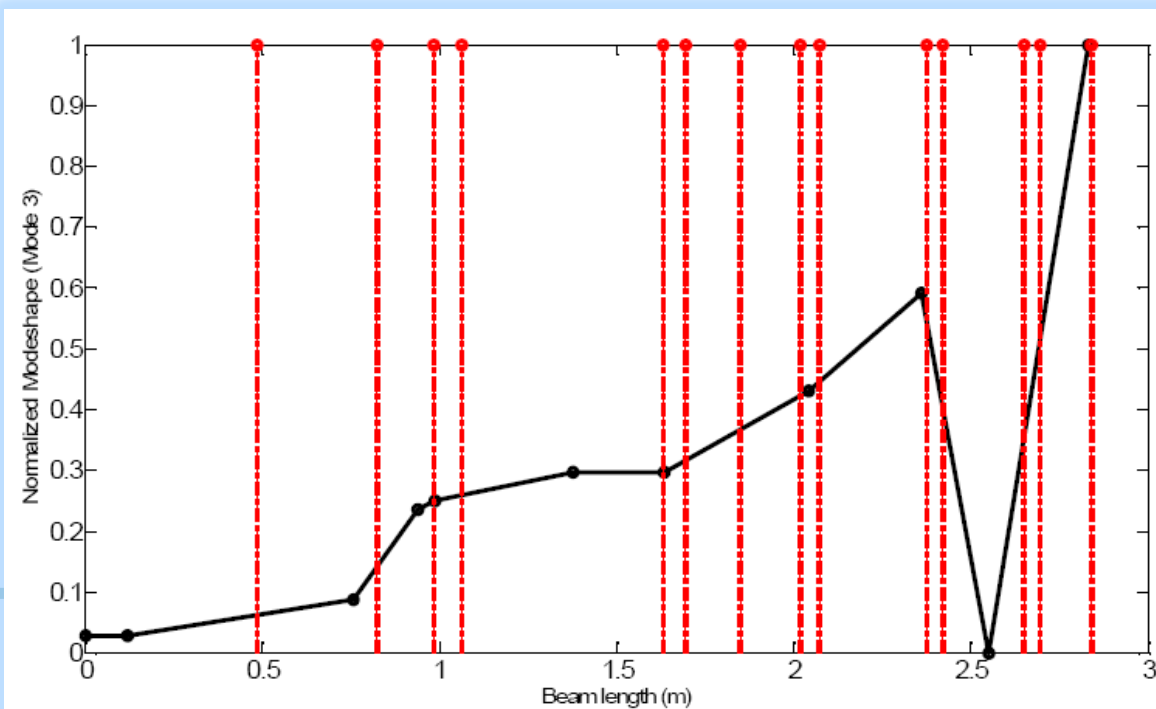
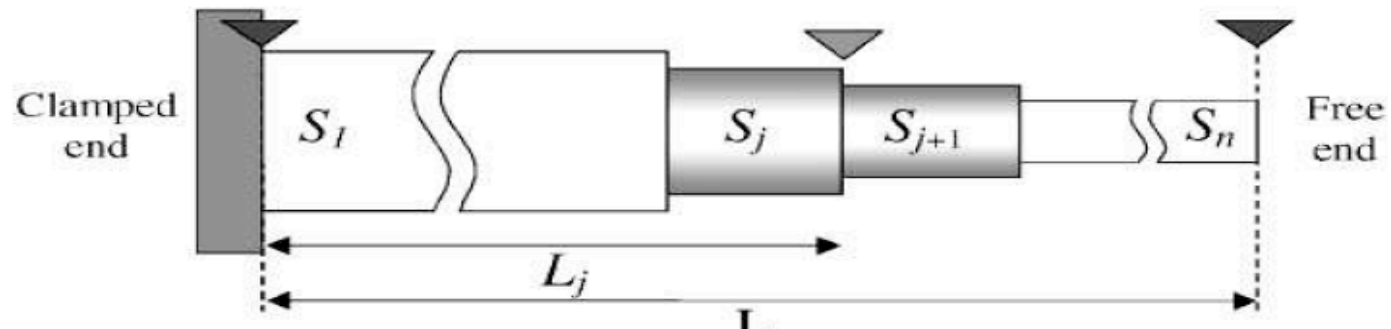


$$Y_i(x) = A_{1i} \cosh(\lambda_i x) + A_{2i} \sinh(\lambda_i x) + A_{3i} \cos(\lambda_i x) + A_{4i} \sin(\lambda_i x)$$





### 3<sup>rd</sup> Mode shape identification: Influence of tapered beam



# Conclusions

- Computer vision methods are able using Lukas-Kanade optical flow algorithm to extract reliable dynamic parameters
- Several validations of the software from simple case to the worst case: Now we know the limitations !
- If small linear displacement hypothesis not checked:
  - Sparse signal reconstruction (Random Sampling)
- Making some important assumptions our method coupled with operational modal analysis could also be used to :
  - Monitor real structure under ambient excitation
  - Check structural integrity

# Future Works

- Multiple camera for 3D dynamic reconstruction (markerless)

