Institut Supérieur de l’Aéronautique et de l’Espace

Estimation of modal parameters confidence intervals:

Application on monitoring of composite beams

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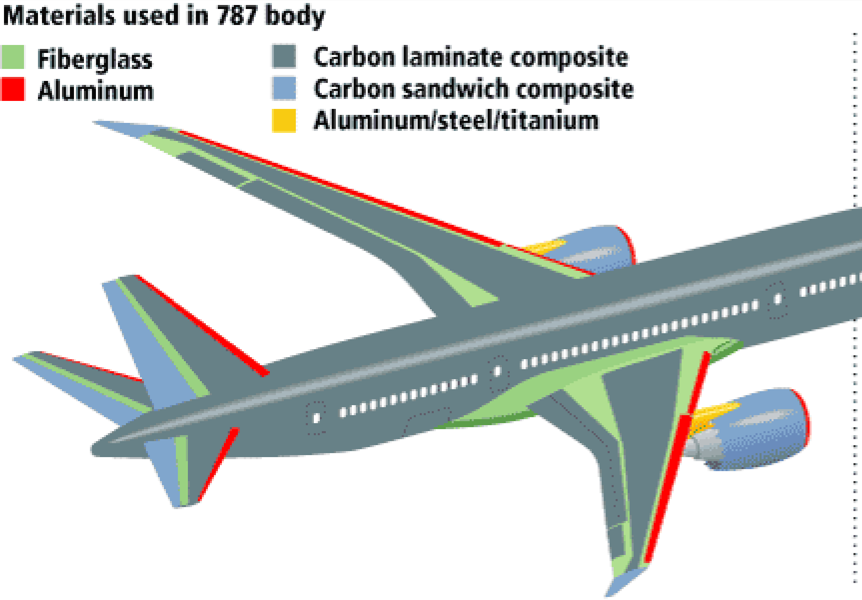
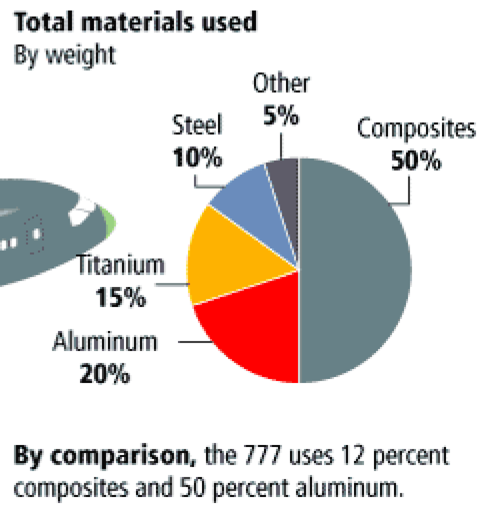
- Objectives
- Modal identification
- Statistical methods
- Results
- Conclusion

SHM

- Introduction
- Application
- Modal indicators

SHM in Smart Structures (Composites)





Boeing 787

50% of Structure

made of Composite

➢ Damage in Composites

- ✓ fabrication stress
- ✓ environmental loadings
- ✓ handling and foreign object impact damage

➢ Aim of Using Composites

- ✓ Increase the specific stiffness and strength
- ✓ Reduce the weight

** Shahdin, Amir and Morlier, Joseph and Gourinat, Yves Correlating low energy impact damage with changes in modal parameters: a preliminary study on composite beams.*

Aim: To provide the condition of a structure.

- **Detection of damage:**
  - ✓ Vibration based → Variations in modal parameters.

Variability in identified modal parameters:

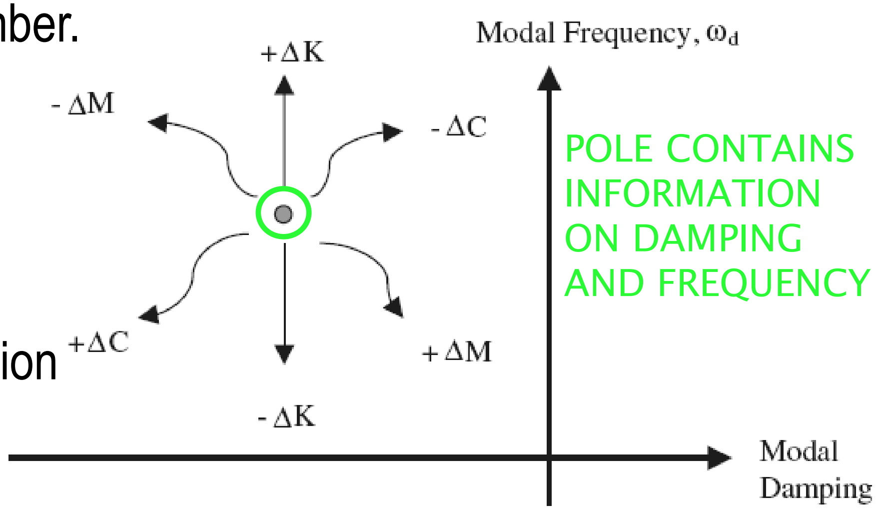
- ✓ May be caused by environmental effects and other systematic sources of variability.
- ✓ May be due to the damage

*A. Rytter: Vibration Based Inspection of Civil Engineering Structures, PhD Thesis, Aalborg University, Denmark, 1993

Damage in a structure changes the modal parameters.

- **Change in natural frequency**
  - ✓ Modes at higher frequency undergo a larger shift
  - ✓ Random with damage due to the symmetric nature of damage.
  - ✓ More evident with increase in mode number.

- **Change in damping:**
  - ✓ Increases with damage due to more friction
  - ✓ Composites show delamination



Adams, R.D., P. Cawley, C.J. Pye and B.J. Stone, "A Vibration Technique for Non-Destructively Assessing the Integrity of Structures" , 1978

Objectives

Objectives of the analysis

➢ Modal parameters extraction

✓ 2 methods:

- Time domain → LSCE
- Frequency domain → UMPA

➢ Characterization of the confidence interval

✓ 2 aims:

- Behaviour of the Methods after noise addition →Analytical FRF
- SHM application → Experimental set of data

We will be able to characterize the variability of modal parameters due to:

✓ **Environmental causes**

✓ **Damage**

Modal parameters

- LSCE & UMPA
- Stabilization diagrams

LSCE & UMPA

LSCE*

$$[h_{rs}(t)] = IFFT\left[\sum_{k=1}^{n}\left(\frac{A_{rs(k)}}{i\omega-\lambda_k}+\frac{\bar{A}_{rs(k)}}{i\omega+\lambda_k}\right)\right]$$

$$\beta_0+\beta_1\lambda_k+\cdots+\beta_{2N}\lambda^{2N}{}_k=0$$

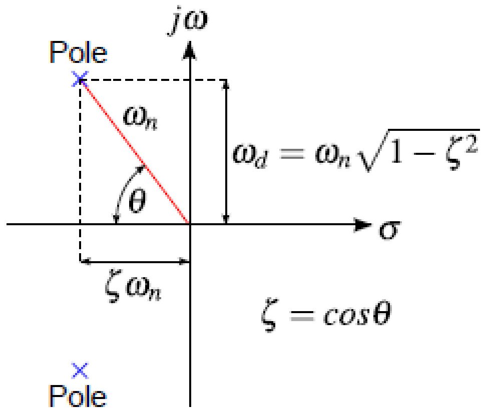
$$D*x=rhs$$

UMPA**

$$[H(\omega)]=\frac{\sum_{k=0}^{num}\left[[\beta_k](j\omega)^k\right]}{\sum_{k=0}^{m}\left[[\alpha_k](j\omega)^k\right]}$$

$$\sum_{k=0}^{m}\left[[\alpha_k](j\omega)^k\right][H(\omega)]=\sum_{k=0}^{num}\left[[\beta_k](j\omega)^k\right]$$

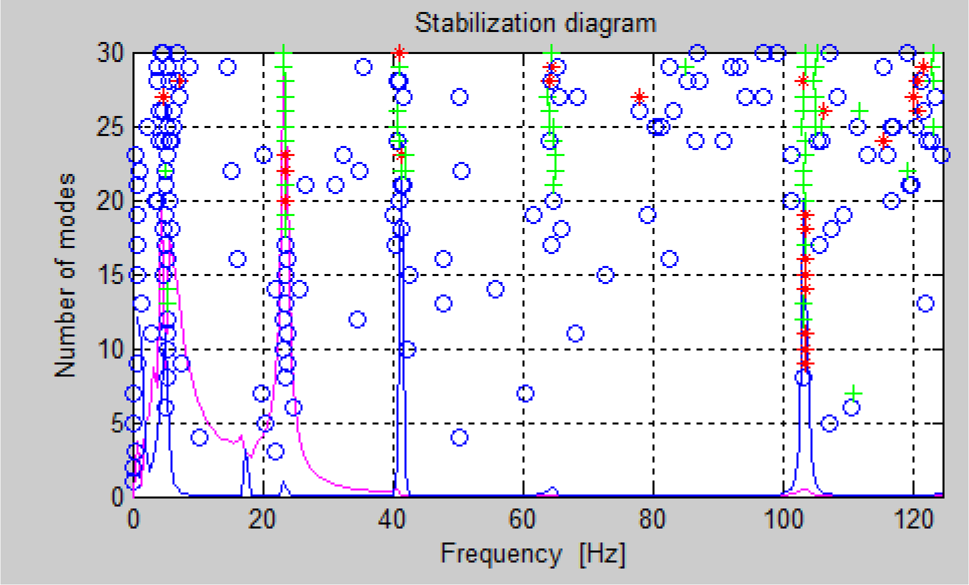
$$x*D=rhs$$

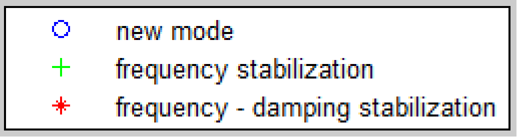


- Kouroussis G., Fekih L.B., Conti C., Verlinden O., EasyMod: A MatLab/SciLab toolbox for teaching modal analysis,International Congress on Sound and Vibration

** Allemang, R.J., Brown, D.L., Fladung W., Modal Parameter Estimation: A Unified Matrix Polynomial Approach, IMAC 1994

Stabilization diagrams





Stabilization criteria : Frequency = 2% , Damping = 5%

Statistical methods

- Jackknife
- Bootstrap

Comparison

> **The original set of data are the n FRFs that have been sampled through experimental testing.**

$$x = \{x_1, x_2, \dots, x_n\}$$

» **Bootstrap**:

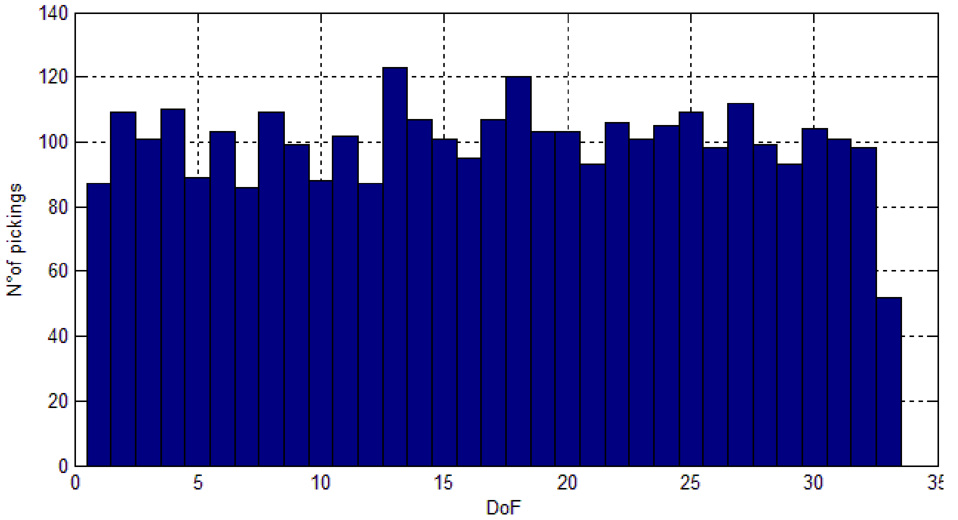
random sampling with replacement

$$\hat{x} = \{\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n\}$$

» **Jackknife**:

leave out iterative process

$$x^{-i} = \{x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n\}$$



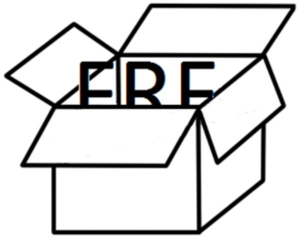
* Bradley Efron, Gail Gong "A leisurely look at the Bootstrap, the Jackknife and Cross-Validation" *The American Statistician*

Comparison

Jackknife	Bootstrap
Similar numerical results	
Gives the same results each time	Gives different results when repeated on the same data
Operational aspects	
Good for verification	Mainly recommended for distribution calculation
Good for basic statistical inference	Intensive computations

Evaluate the quality of the identified modal parameters: assess the confidence.

N	CI	$\bar{x}_i$
50	95%	$0{,}78\sigma + \bar{x} \leq \bar{x}_i \leq \bar{x} + 1{,}25\sigma$
100	95%	$0{,}88\sigma + \bar{x} \leq \bar{x}_i \leq \bar{x} + 1{,}16\sigma$

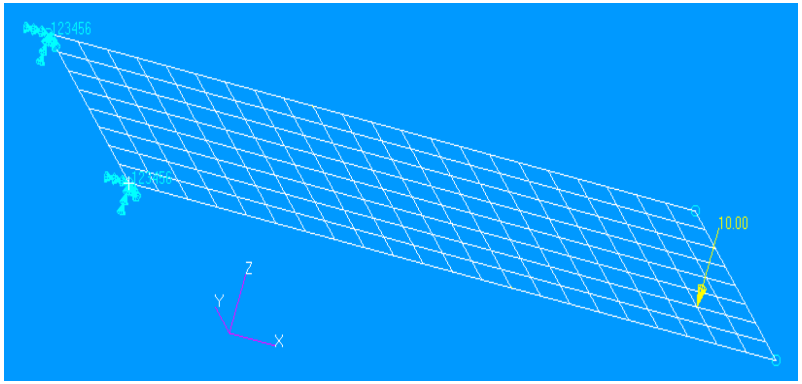


** Randall J. Allemang, «Modal Analysis-Where Do We Go From Here?», IMAC 1993

Analytical test
(Supervised data)

- Simulation setup
- Effect of noise

Simulation model



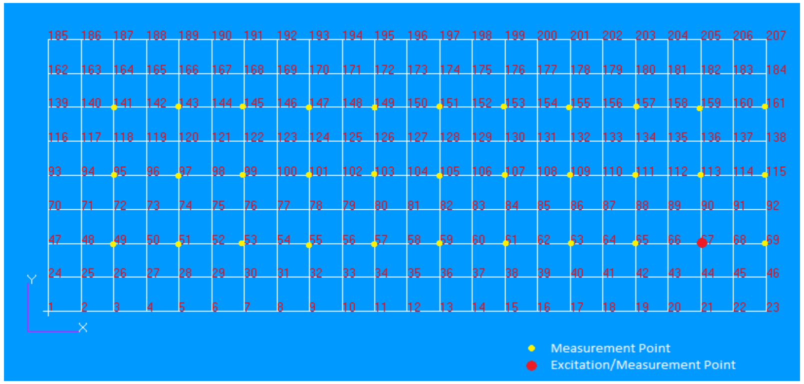
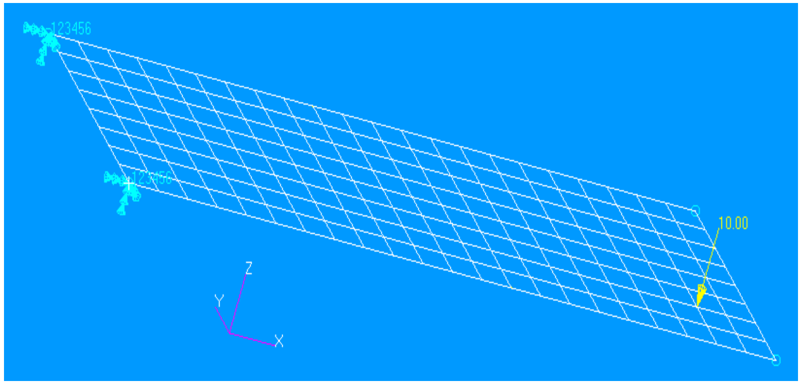


Plate clamped at one side excited so to extract torsional and bending modes

Meshed → stable modal frequencies

	ω_n [Hz]	$\omega_{n\text{-mean}}$ [Hz]			
		BS	**JK**	**BS**	**JK**
N°Mode	**FEM**	**LSCE**	**LSCE**	**UMPA**	**UMPA**
1	45.05	45.06	45.06	45.08	45.10
2	281.32	281.32	281.32	281.27	278.50
3	416.58	416.58	416.58	416.53	416.57
4	787.17	787.17	787.17	787.17	787.14
5	1272.73	1272.94	1275.14	1272.73	1272.60

Simulation model



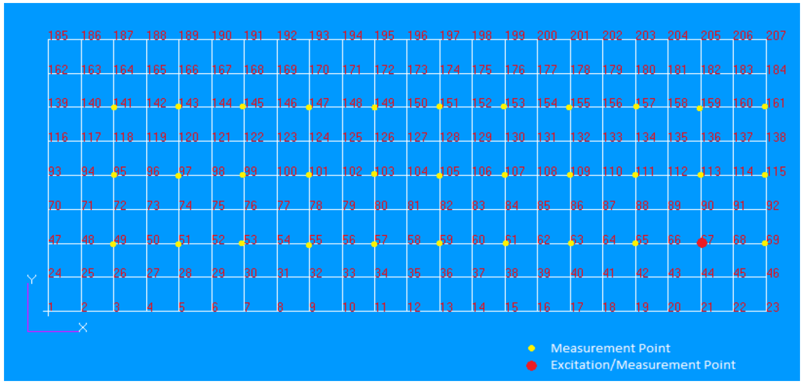
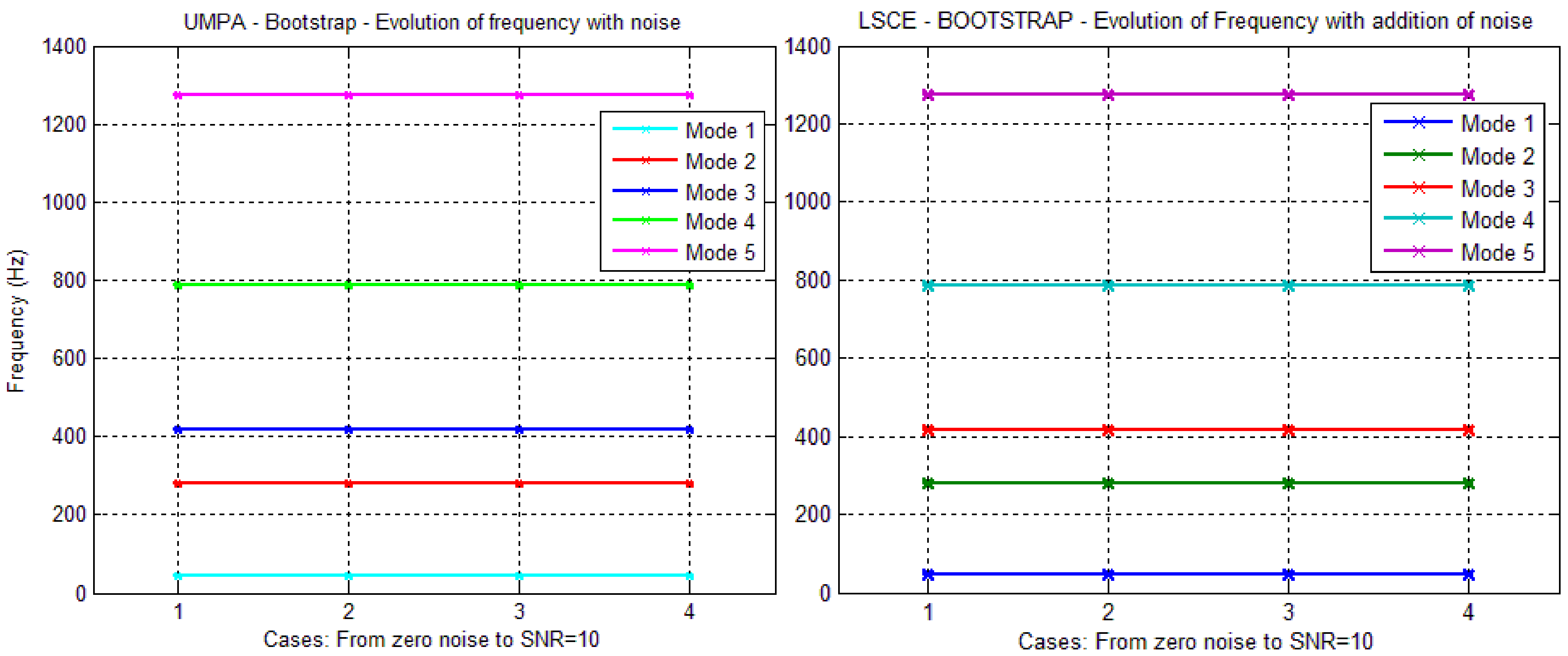


Plate clamped at one side excited so to extract torsional and bending modes

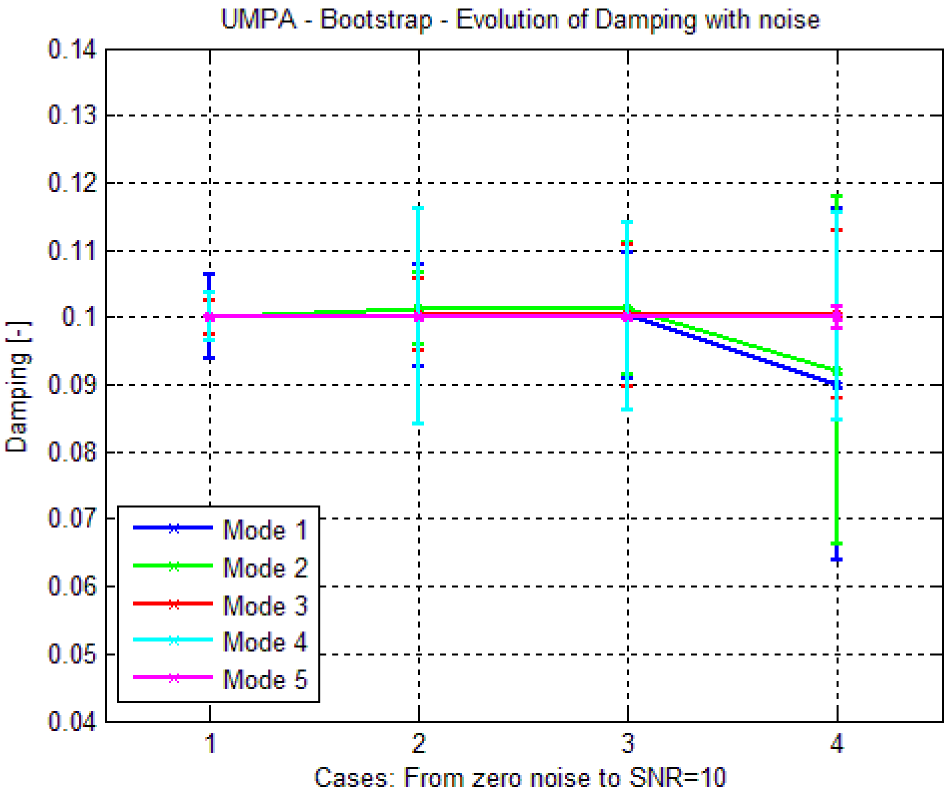
Meshed → stable modal frequencies

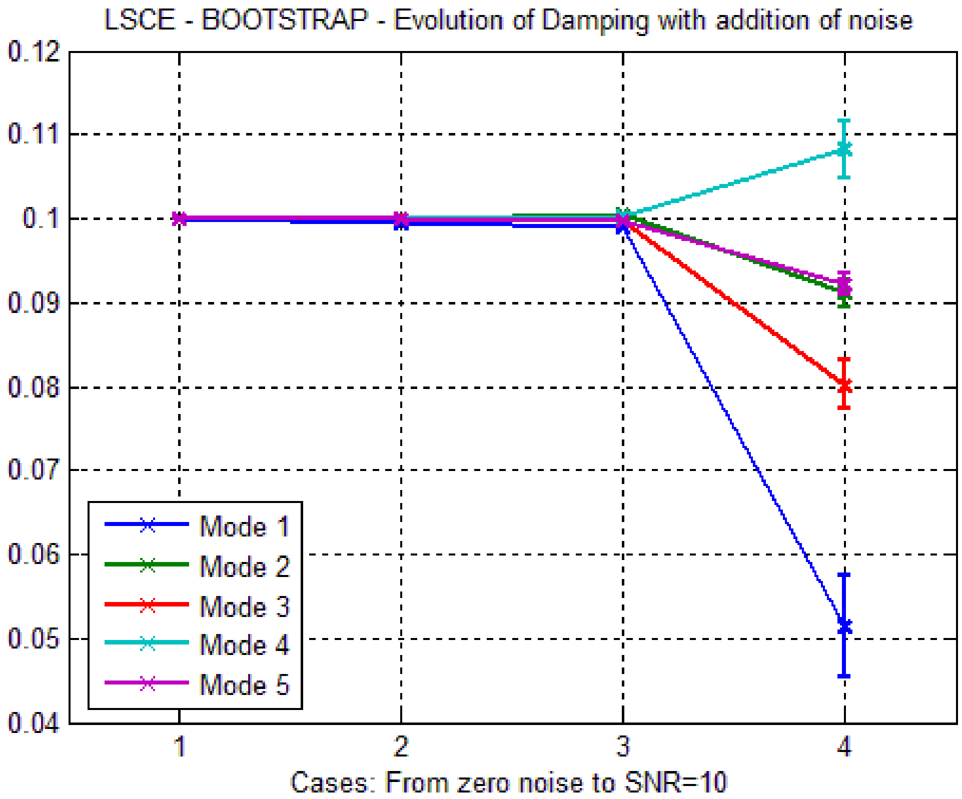
	ζ							
	No Noise		SNR = 100		SNR = 50		SNR = 10	
N° mode	LSCE	UMPA	LSCE	UMPA	LSCE	UMPA	LSCE	UMPA
1	0.100	0.100	0.100	0.1003	0.099	0.1003	0.051	0.0900
2	0.100	0.100	0.100	0.1013	0.100	0.1013	0.091	0.0920
3	0.100	0.100	0.100	0.1003	0.100	0.1003	0.080	0.1003
4	0.100	0.100	0.100	0.1000	0.100	0.1000	0.108	0.1000
5	0.100	0.100	0.100	0.1000	0.100	0.1000	0.092	0.1000

Frequency change after noise addition



Damping change after noise addition

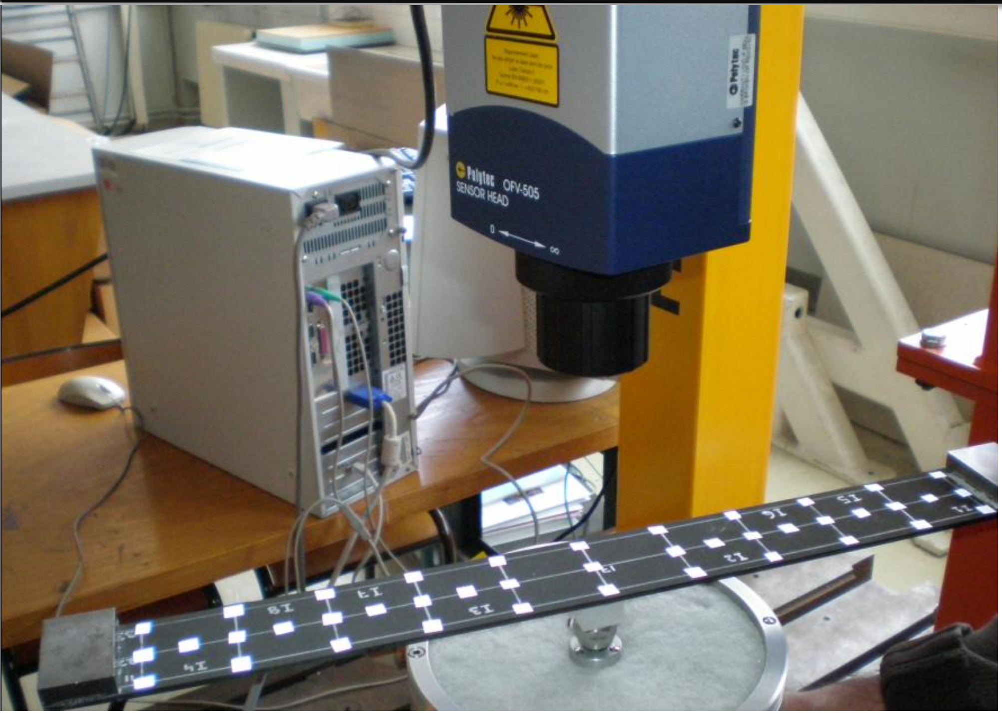




Experimental test

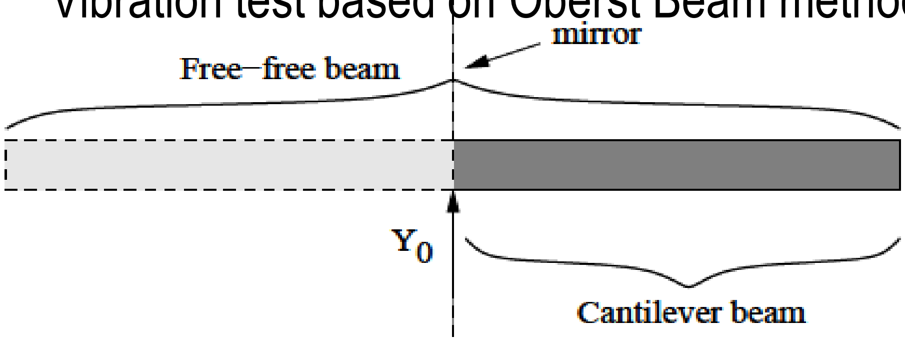
- Experimental setup
- Damage detection
- Results

Test setup



Type of material	T300/914
Dimensions of the beam	480 x 50 x 3 mm
No of plies	24
Lay-up	[0/90/45/-45]3s

Vibration test based on Oberst Beam method



- Acquisition Parameters

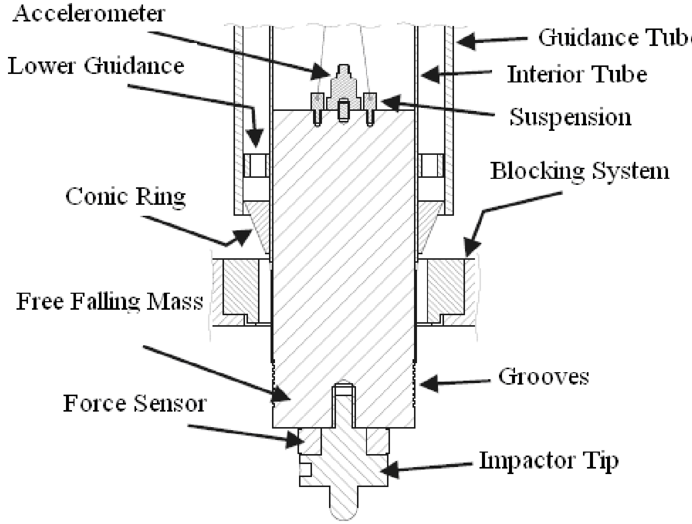
✓ Frequency Resolution = 0.25Hz
✓ Excitation level = 1N

** Shahdin, Amir and Morlier, Joseph and Gourinat, Yves Correlating low energy impact damage with changes in modal parameters: a preliminary study on composite beams, 2009*

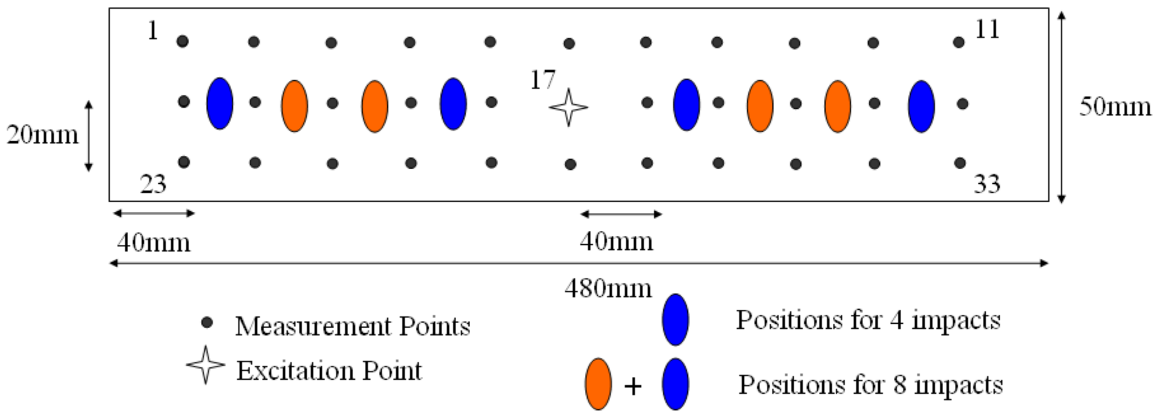
Impact Test

- > **Drop weight impacts**
- > **BVID (below the limit)**
  - » Effectiveness of vibration testing
- > **Detectability (0.6 mm)**

Drop tower

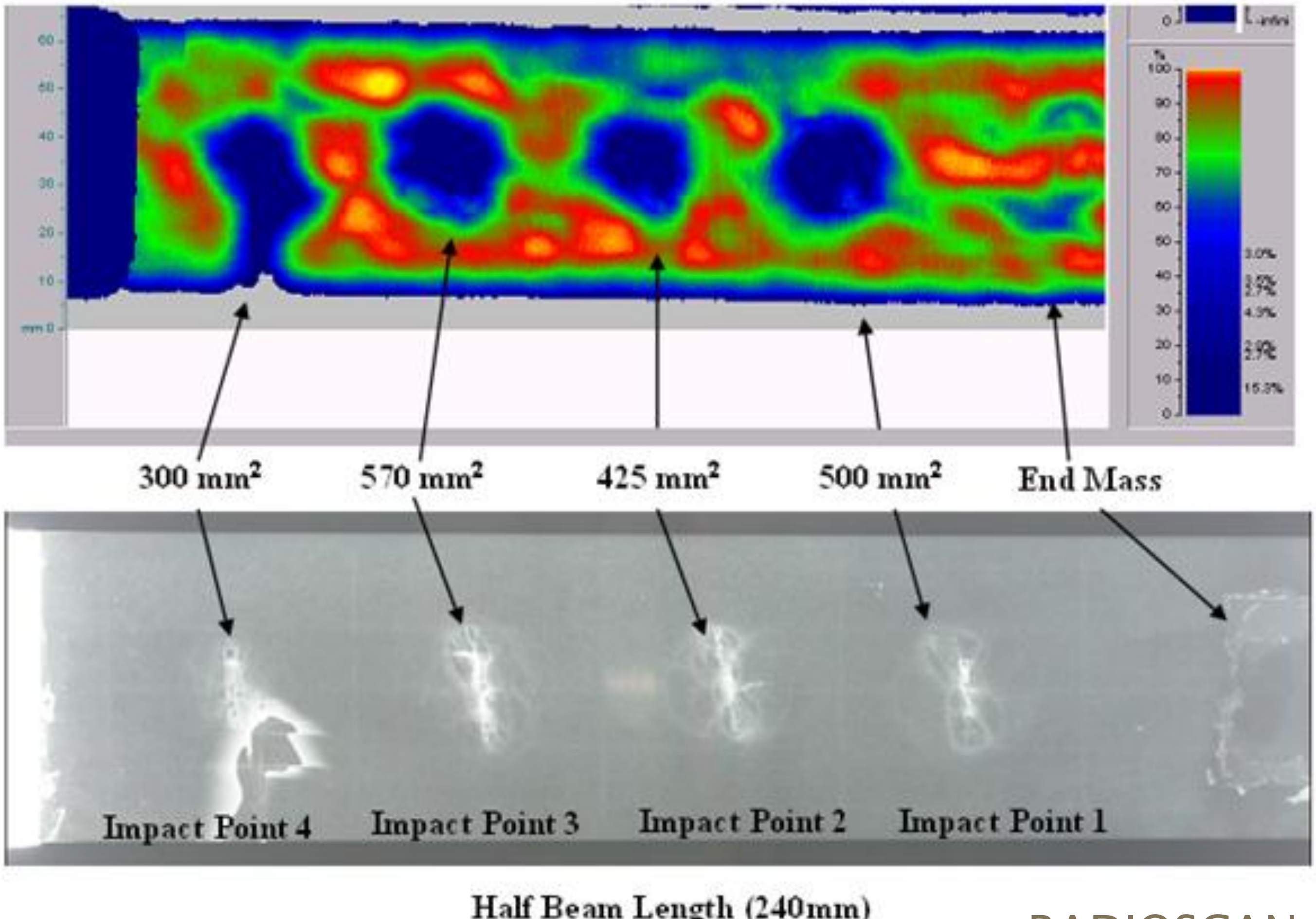


t [mm]	E_{impact} [J]	H [mm]	V_{impact} [m/s]
3,12	7	387	2,64



NDT results for impact tests (repetitivity)

C_SCAN



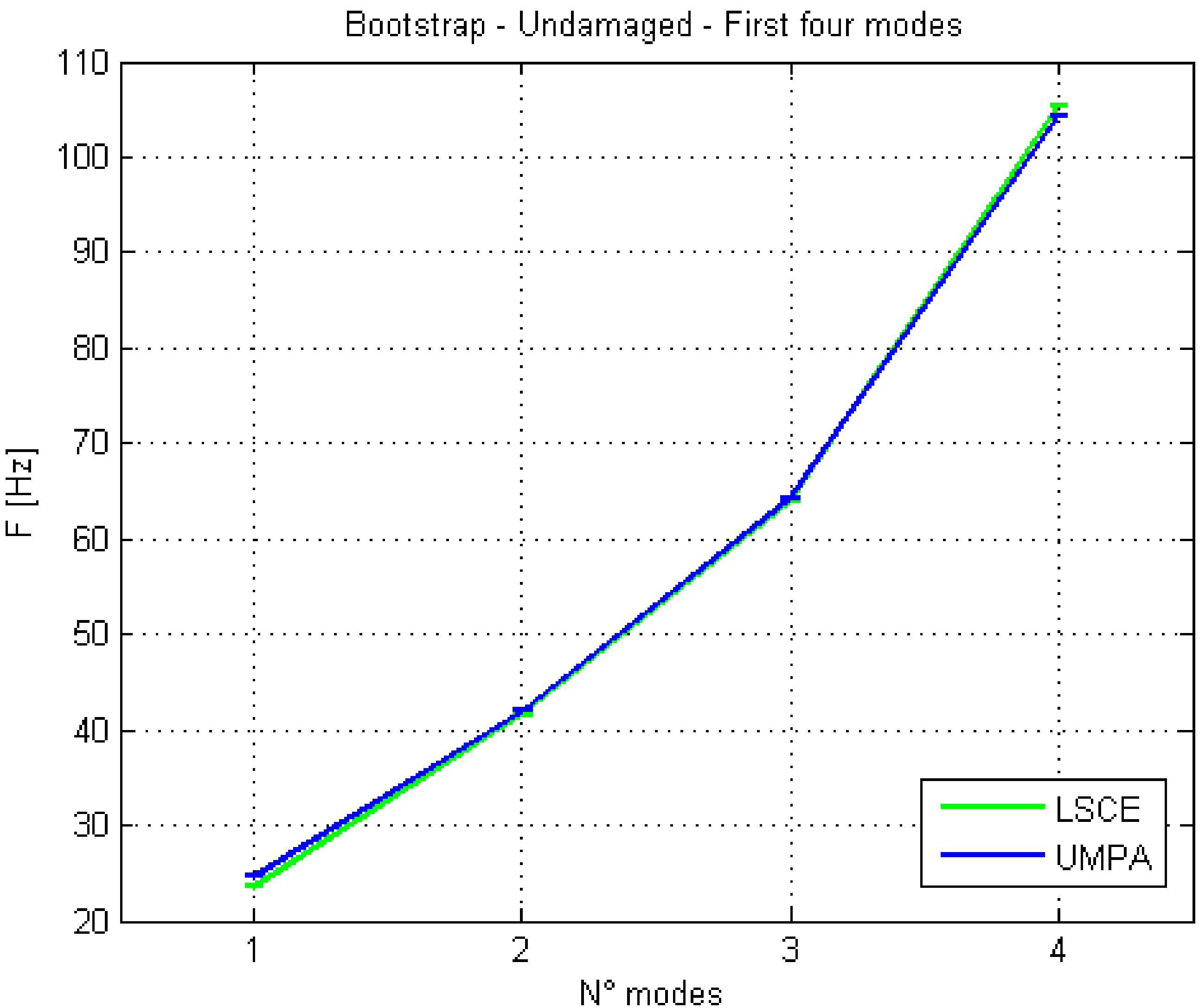
RADIOSCAN

Damage zones:
local loss of rigidity
(decrease in frequency)

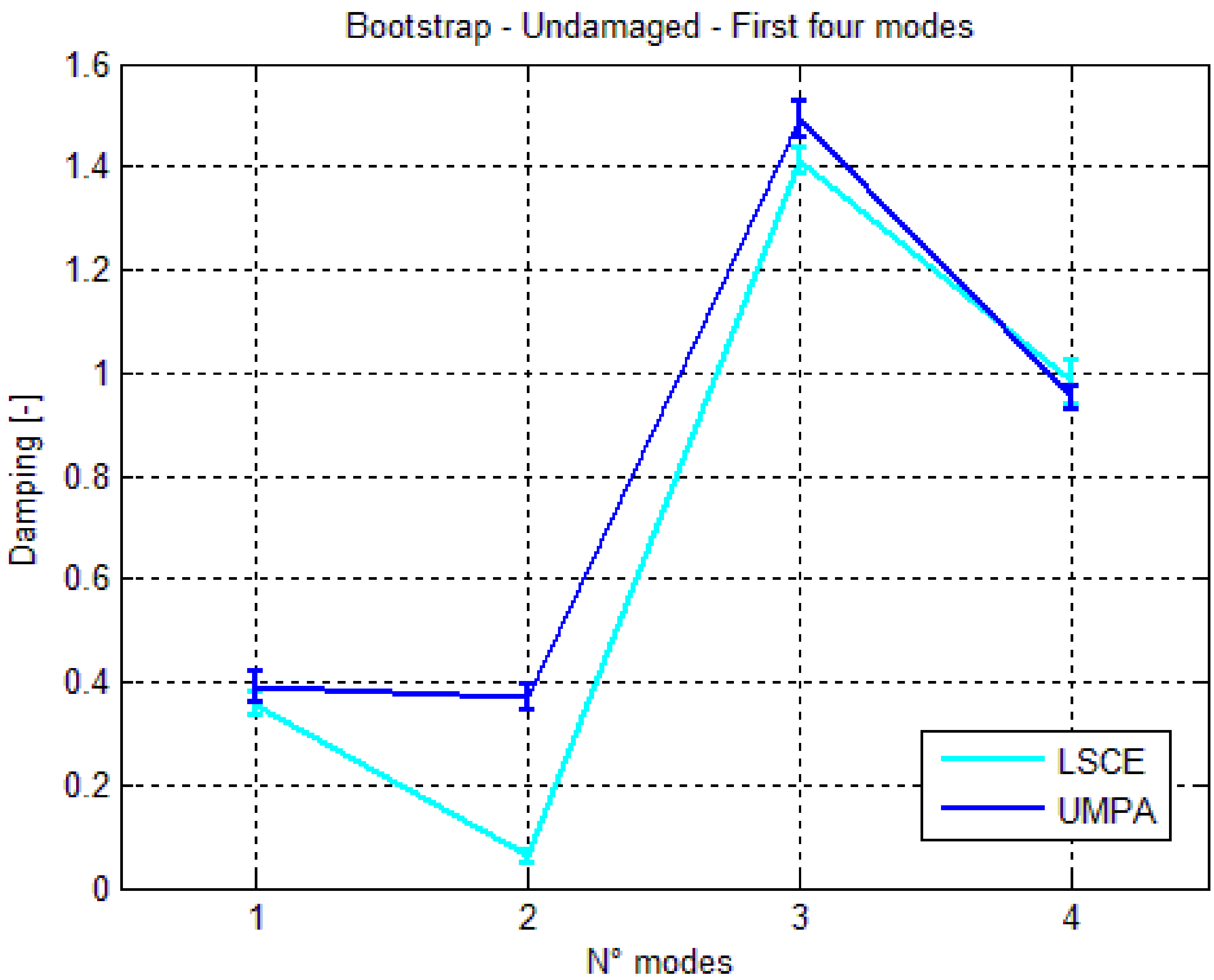
and

increase the surface of friction
(increase in damping)
due to delamination

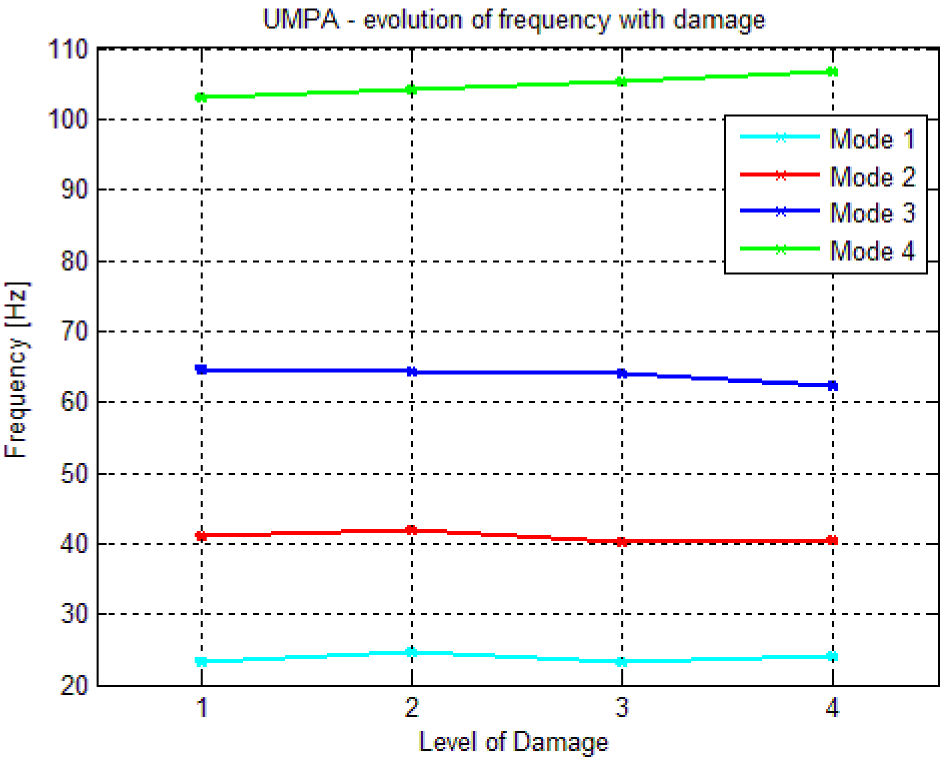
UMPA & LSCE – Natural Frequencies

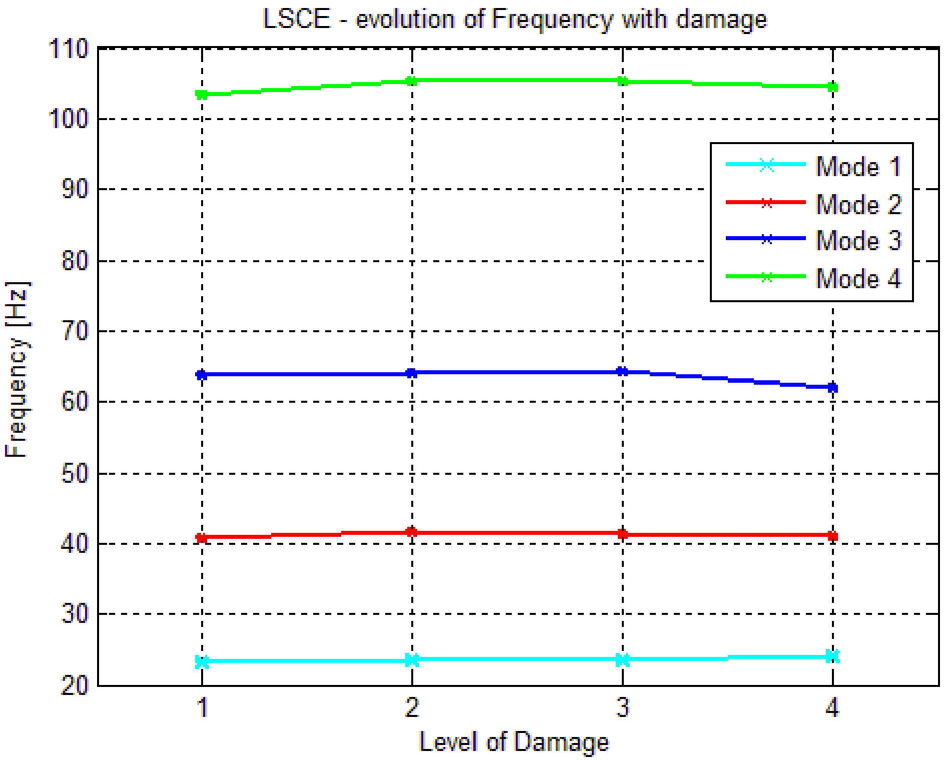


UMPA & LSCE – Damping ratio



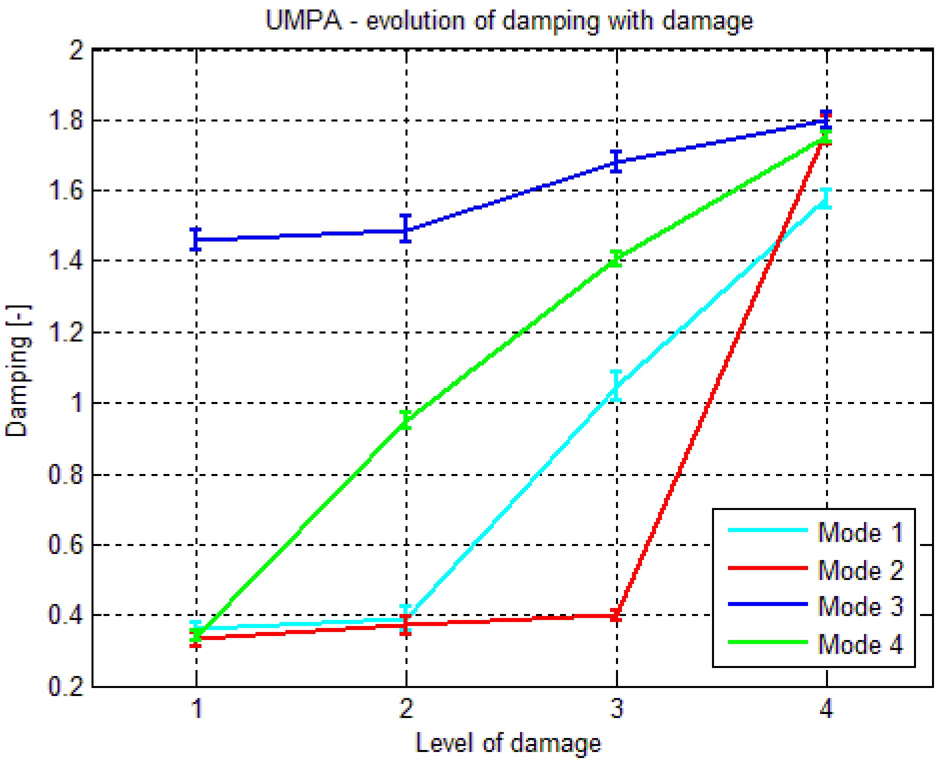
Natural frequencies after progressive damage

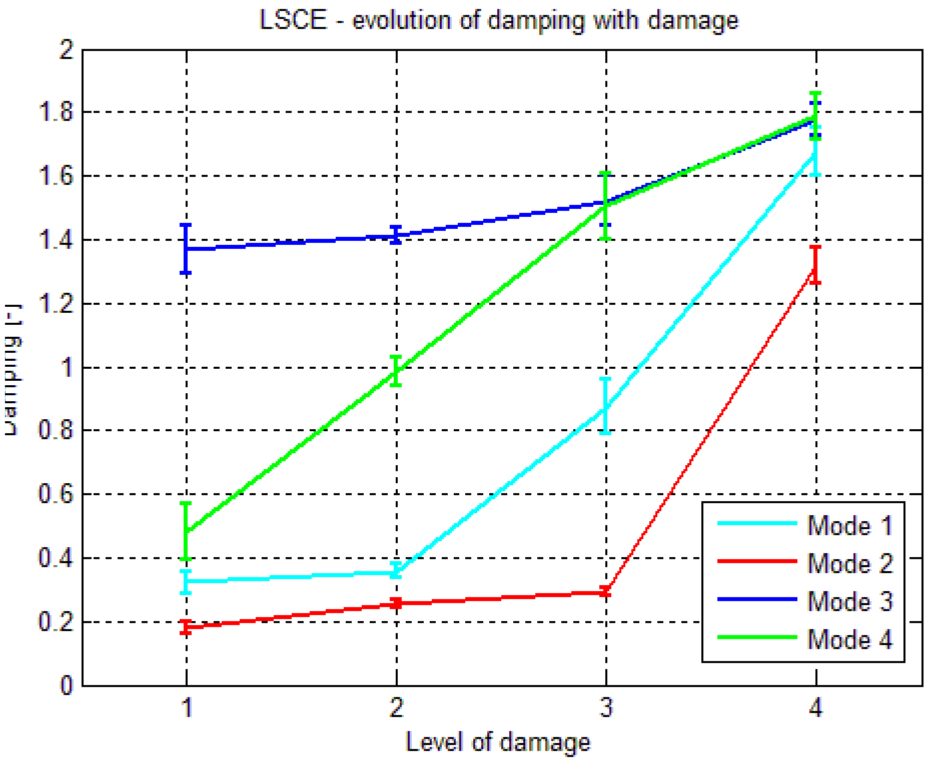




	σ_ω			
	LSCE		UMPA	
	Min	Max	Min	Max
Order of magnitude	6.00E-03	8.00E-02	6.00E-03	6.00E-02

Damping after progressive damage





	σ_ζ			
	LSCE		UMPA	
	Min	Max	Min	Max
Order of magnitude	6.00E-03	1.00E-01	1.00E-02	3.00E-02

Conclusions

- **Regarding the Methods for modal analysis**
  - ✓ UMPA robust respect to noise and is more reliable to extimate the damping ratio
  - ✓ LSCE is a very quick method, good for the estimation of natural frequency

- **Regarding the statistical methods**
  - ✓ Bootstrap takes into account both bias and random error, better estimation of the worse case
  - ✓ Jacknife is a quick tool

- **Regarding SHM**
  - ✓ Damping is a good index for damage at low frequency

Thank you ☺

Questions?