IMAC-XXVII

Original Statistical Approach for the Reliability in Modal Parameters Estimation

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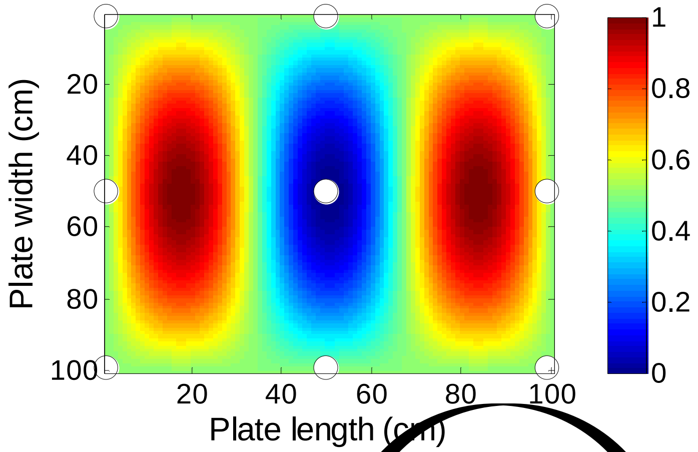
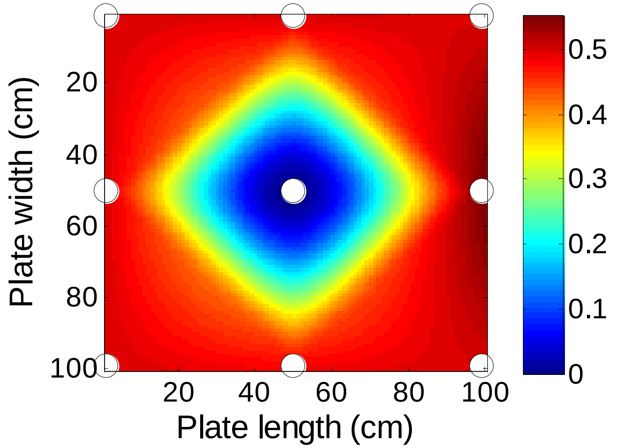
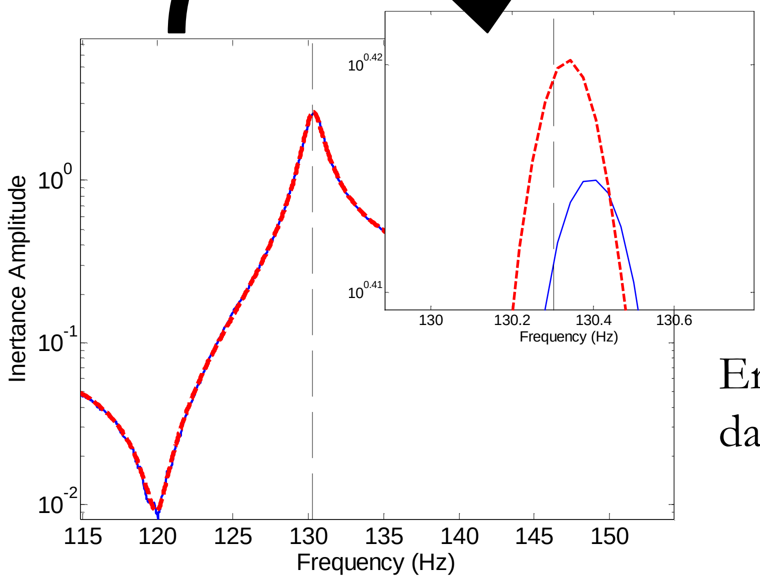
SUPAERO

Goal: Study of the reliability of damping estimation and mode shape reconstruction using supervised data.

(2,1) mode shape reconstruction from a 9 points regular grid **> spatial aliasing**







Error in pole estimation affects frequency, damping and mode shape estimation

Introduction

Theoretical background

Methods/Results for mode shape reconstruction

Benchmark for damping estimation

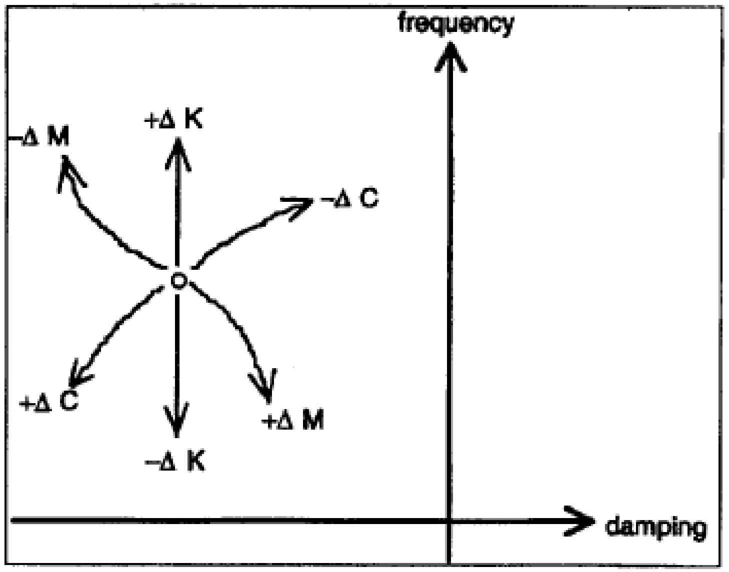
Conclusions

Problem statement & Motivation

Deciding on an optimal sensor placement and optimal frequency sampling is a critical issue in the construction and implementation of an effective Structural Health Monitoring system.

This paper focuses on the estimation of damping ratio because among the modal parameters, it is the most difficult to model/estimate. **Damping is used to detect delamination (friction zone) in composites (aeronautical structures)**

Reliable modal parameters estimation **> Robust SHM algorithms > automatic diagnosis**

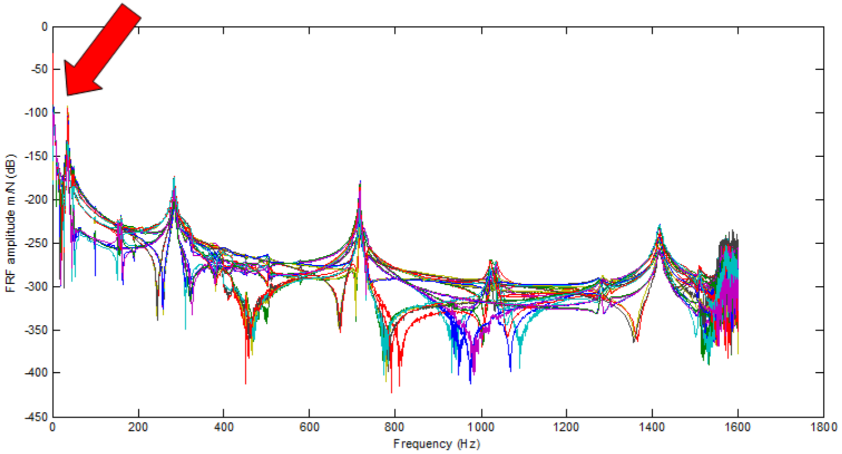


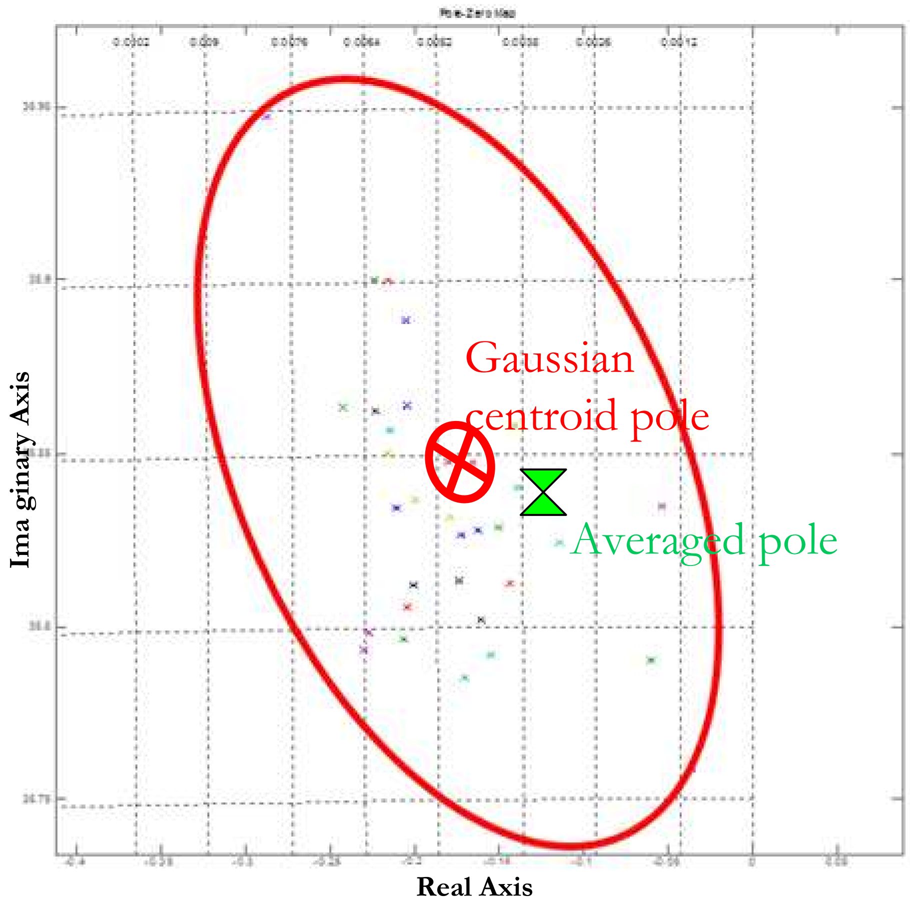
Movement of a Pole Due to Mass, Stiffness, & Damping Changes

Tracking of Poles for Damage Detection

Modal parameters estimation is dependent of the algorithm used

First mode of 33 FRFs of composite beams T300-914





- Stability in Frequency :
  Imaginary part varies from 35.8 to 35.95 Hz
- Damping variability from 0.1% to 0.8%

- PLSCE estimated frequency=35.91Hz
  PLSCE estimated damping= 0.585%

- RFP gives close results but slighty different (damping)

Who is the more accurate ?? > supervised Benchmark

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RFP, Rational Fraction Polynomial

frequency domain method

1. FRF estimation in a ratio of 2 polynoms

$$H(\omega)=\frac{\sum_{k=0}^{2N-1} a_k (i\omega)^k}{\sum_{k=0}^{2N} b_k (i\omega)^k}$$

with :

b_k permits to extract poles **> frequencies and damping ratios**

a_k Residue **> Mode shapes**

$$=\sum_{k=1}^{N} \frac{A_{rs(k)}}{(i\,\omega-\lambda_k)}+\frac{A^*_{rs(k)}}{(i\,\omega-\lambda^*_k)}$$

2. a_k and b_k computation by minimizing E the error between experimental and analytical FRF by least square estimation.

$$E=\sum_{\omega=\omega_{min}}^{\omega_{max}} \left| \underbrace{H_{rs}(\omega)}_{\text{Theoretical value}} - \underbrace{H^{EXP}_{rs}(\omega)}_{\text{Measured data}} \right|^2$$

CWT,
Continuous Wavelet Transform

mixed time-frequency approach

Wavelet coefficients of the Impulse Response Function:
time-frequency map computation:

$$Wf(u,s) = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{s}} \psi^*(\frac{t-u}{s}) dt$$

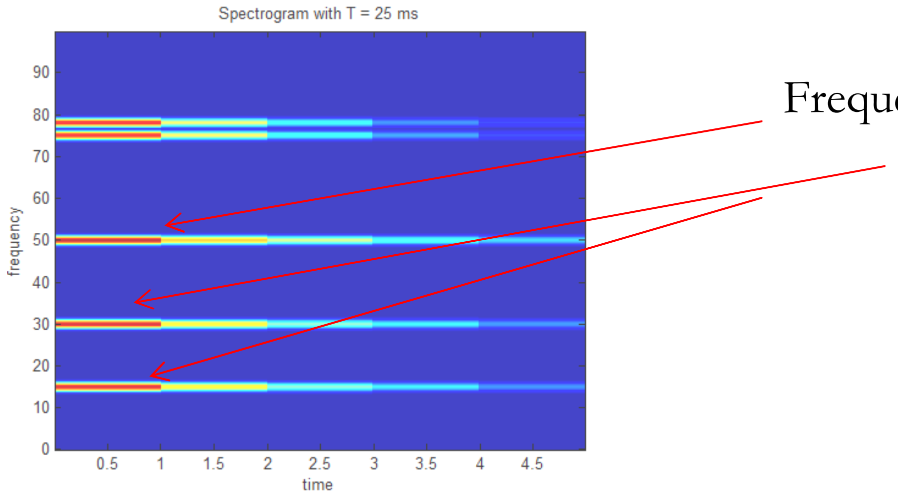
- u translation; - s scale

using Morlet's wavelet :

$$\psi(t) = e^{j\omega_o t} e^{\frac{-t^2}{2}}$$

Preprocessing

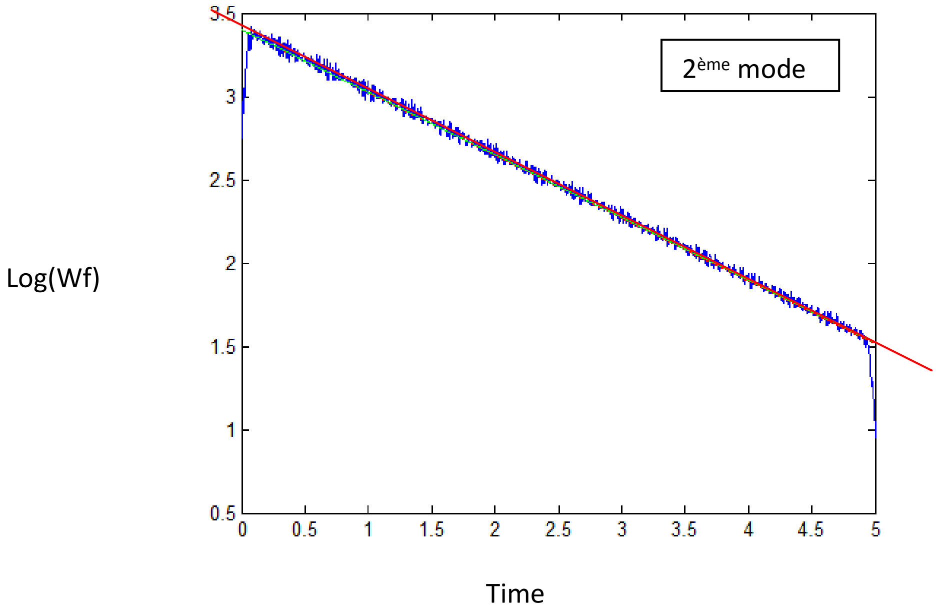
Maxima lines extraction using Short Time Fourier Transform map



Damping estimation

For each maxima line (each resonant frequency) we plot the log of wavelets coefficients Wf function of time

$$\ln\left|(w_g x)(a_0,b)\right| \approx -\zeta w_n b + \ln\left(A_0\left|G^*\left(\pm i a_0 w_n \sqrt{1-\zeta^2}\right)\right|\right)$$



LSCE, Least Square Complex Exponential

time domain method

Global estimation of natural frequency and damping of several modes simultaneously

Taking the inverse Fourier Transform, the expression for an Impulse Response Function (IRF) in terms of modal parameters is given as:

$$h_{rs}(t) = \sum_{k=1}^{N} A_{rs(k)}\, e^{\lambda_k t} + A^*_{rs(k)}\, e^{\lambda^*_k t} = 2\,\mathrm{Re}\left(\sum_{k=1}^{N} A_{rs(k)}\, e^{\lambda_k t} \right)$$

$Z_k = e^{\lambda_k \Delta t}$ defines the natural frequency and damping ratio of mode k (= global estimate)

$A_{rs(k)}$ defines the displacement of point r in mode k (= local estimate)

LSCE,
Least Square Complex Exponential

time domain method

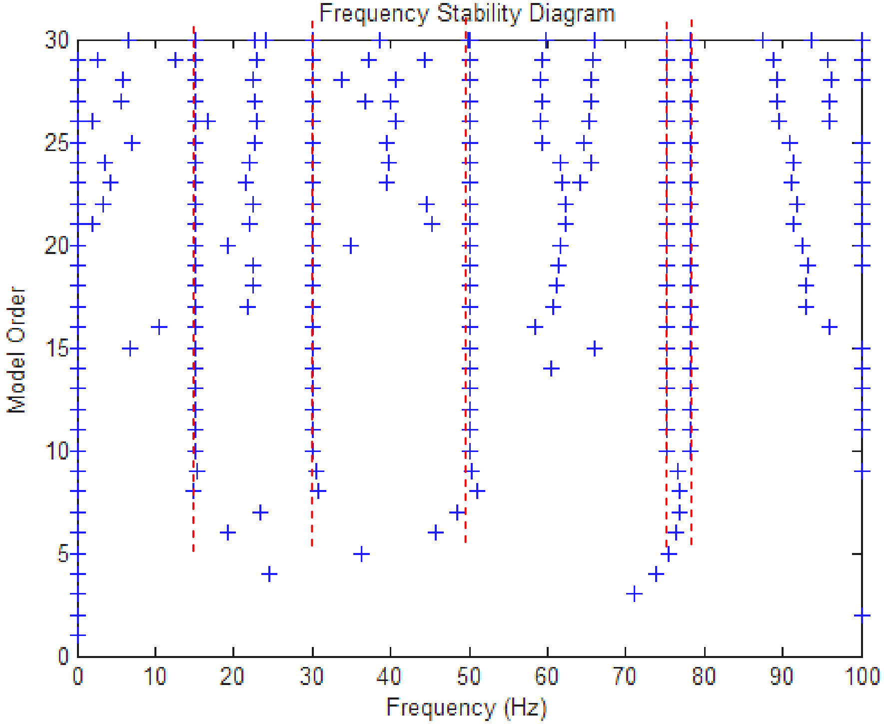
The unknowns Z_k may be considered as the roots of a polynomial of order $2N$:

$$P(Z) = \prod_{\substack{k=-N \\ k\neq 0}}^{N} (Z - Z_k) = \sum_{p=0}^{2N} \alpha_p Z^p$$

where the coefficients α_p have to be estimated using data measured on the system.

By using a different number of poles ($2N$) and comparing the error between the regenerated FRF's and the original measured data, it is possible to draw a so-called '**stability diagram**':

As the number of poles increases, a number of computational modes are created in addition to the genuine physical modes which are of interest.



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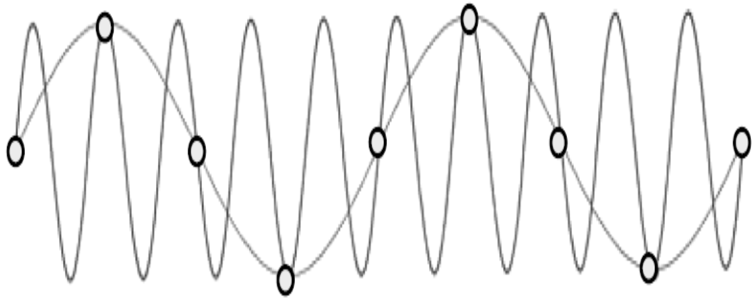
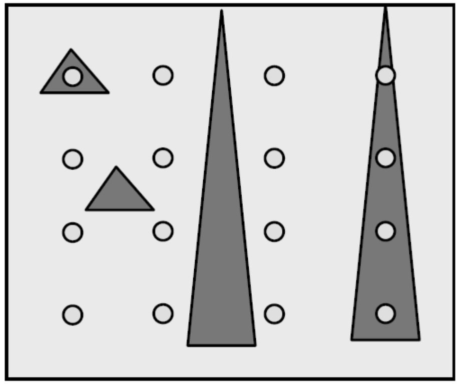
Aliasing in 2D

Mapping a continuous function to a discrete one is called sampling.



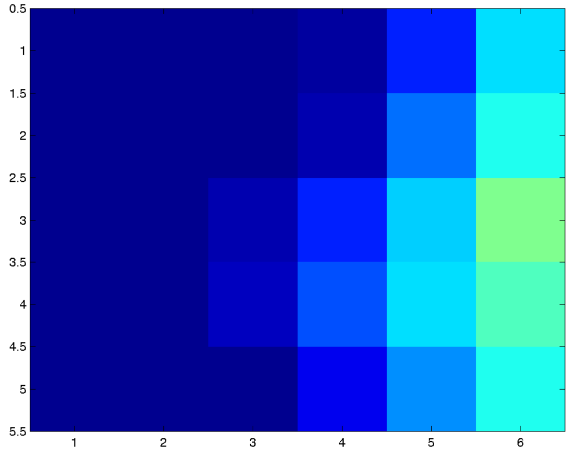


In general artefacts are due to under-sampling or poor reconstruction:
Temporal aliasing (Shannon's theorem) (a), Spatial aliasing (b)
due to limited spatial resolution and induce loss of details.

Mapping in 2D

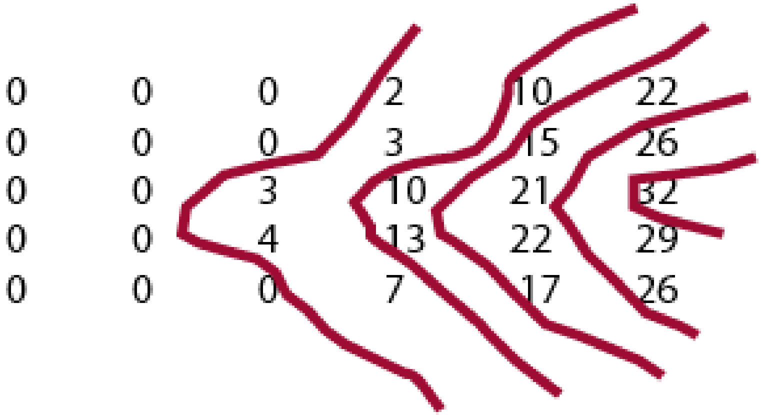
Exemple:
a set of numbers

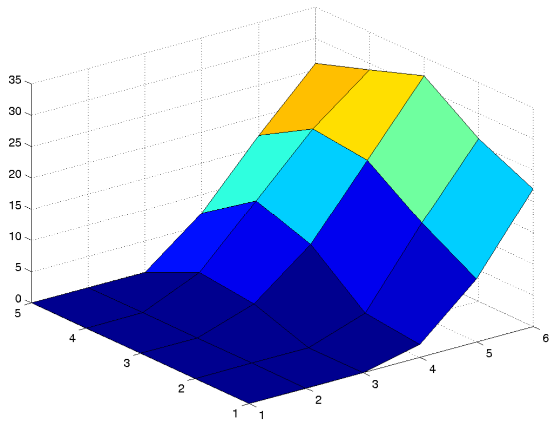
0	0	0	2	10	22
0	0	0	3	15	26
0	0	3	10	21	32
0	0	4	13	22	29
0	0	0	7	17	26



Color shading

hand

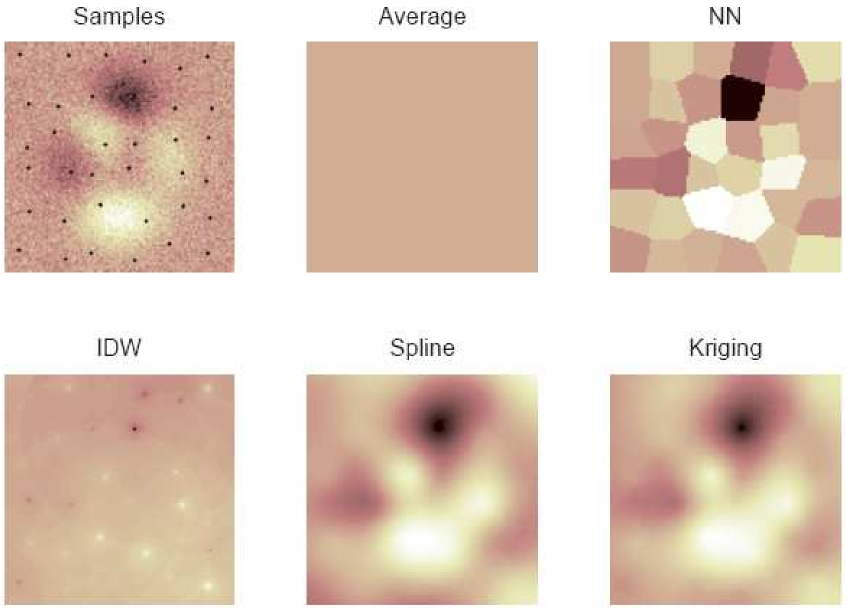




3D perspective

Mapping in 2D

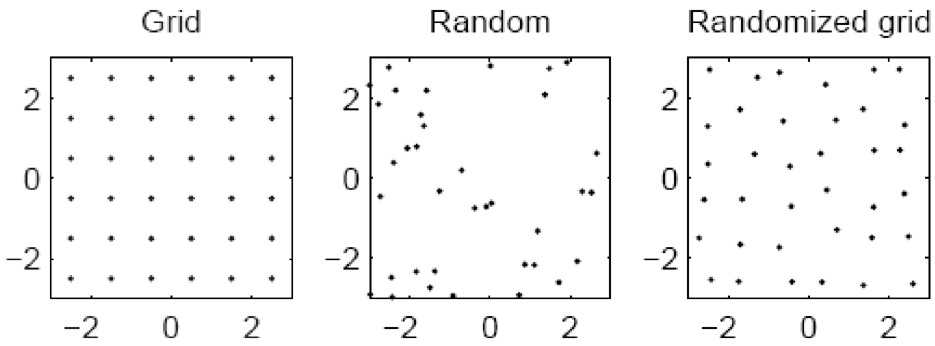
Interpolation = surface reconstruction



Several different sampling and interpolation methods have been tested and compared using simulated data **[25]. Uniform grid sampling proved to be superior** to both random sampling and cluster sampling.

Spline and kriging interpolation gave good results from a certain sampling density and **upwards, kriging being slightly the best.**

The peaks surface from Matlab was used to generate a test map. This smooth surface, has two valleys and three peaks, was added some random Gaussian noise in order to simulate the local variance or sampling noise

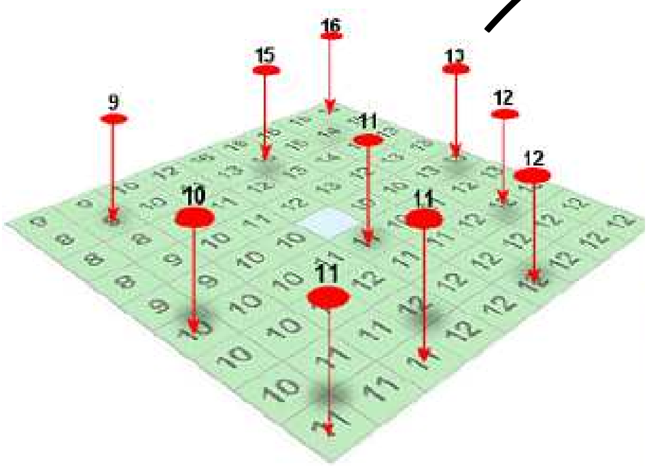


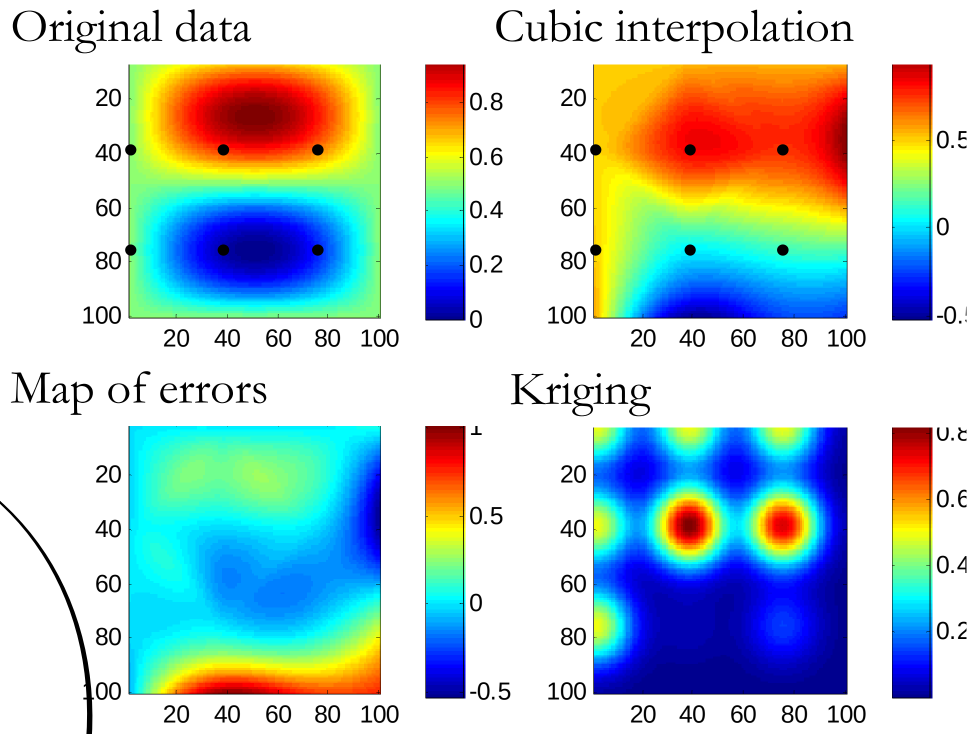
[25] Kirk K., Spatial sampling and interpolation methods - comparative experiments using simulated data, Master's thesis, Aalborg University, 2003.

Regular grid

Modeshape reconstruction=2D interpolation

The primary assumption of spatial interpolation is that points near each other are more alike than those farther away; therefore, any location's values should be estimated based on the values of points nearby.





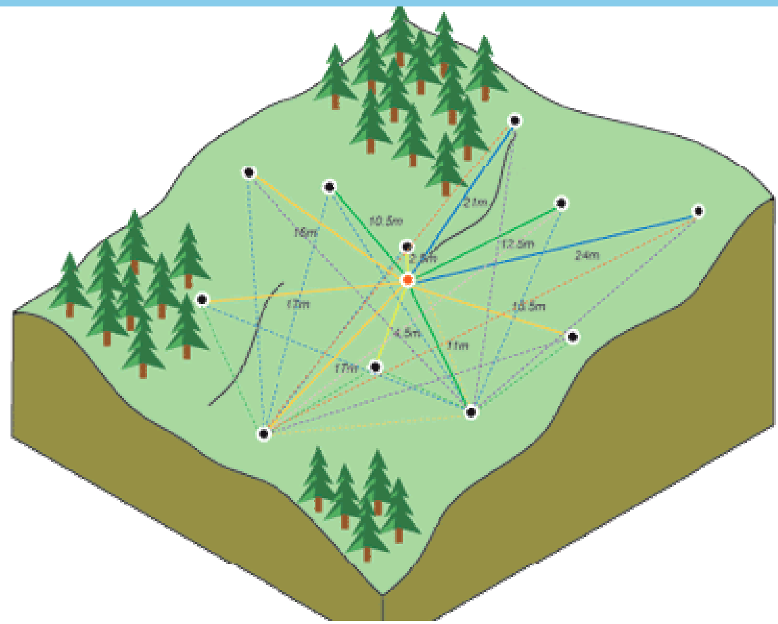
Mean Absolute Error (MAE)

Optimal sensors placement
for reliable mode shape reconstruction
> Damage Detection Algorithms

Kriging

statistical models that include autocorrelation and consider a trend in the data

Kriging is a group of geostatistical techniques to interpolate the value of a random field (e.g., the elevation, *z* of the landscape as a function of the geographic location) at an unobserved location from observations of its value at nearby locations



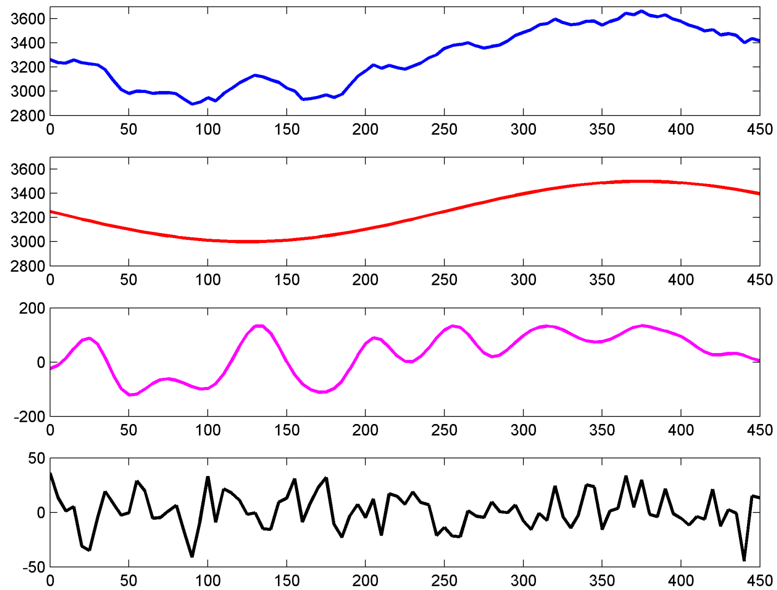
value of z at location s modeled as sum of:

trend (m)

spatial autocorrelation (ε')

Gaussian error (ε'')

$$Z(s) = m(s) + \varepsilon'(s) + \varepsilon''(s)$$

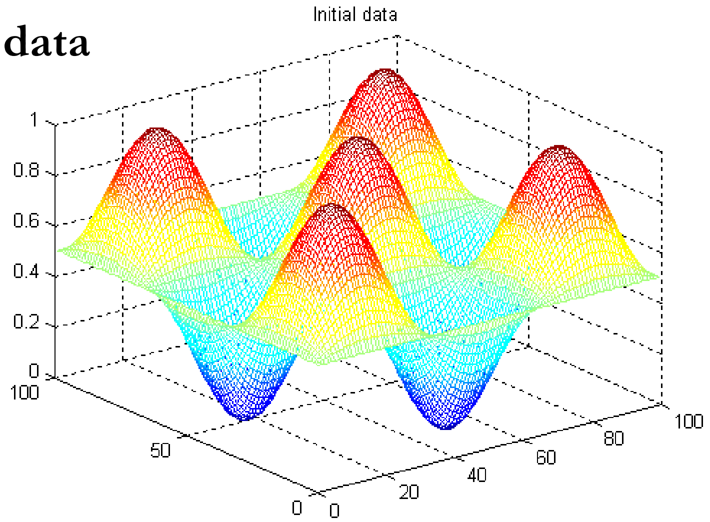


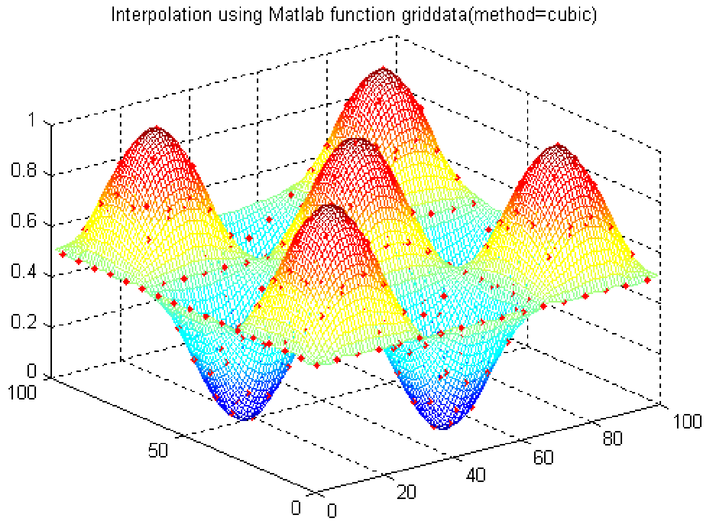
Regular grid

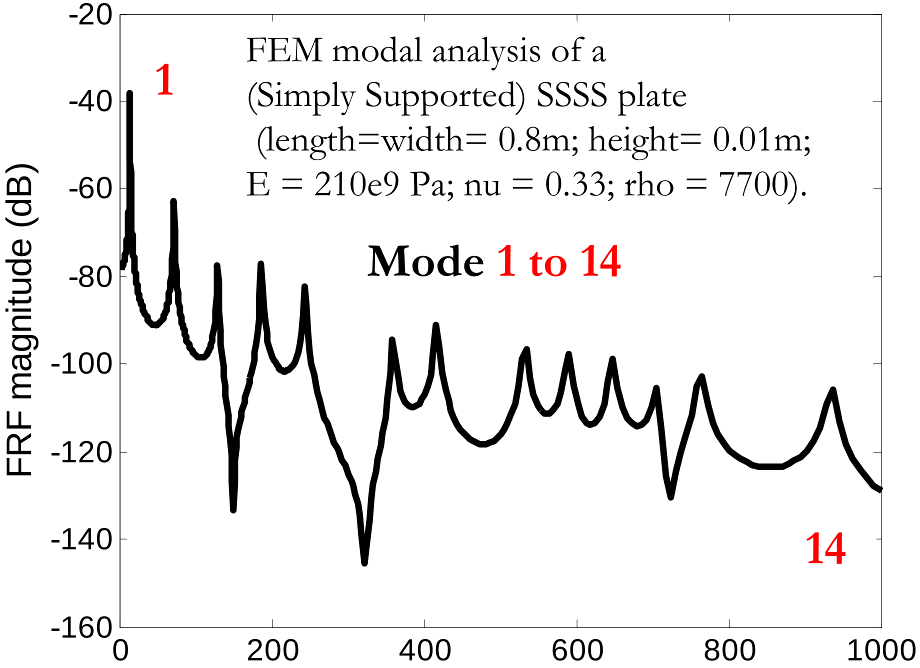
Modeshape reconstruction=2D interpolation

original data

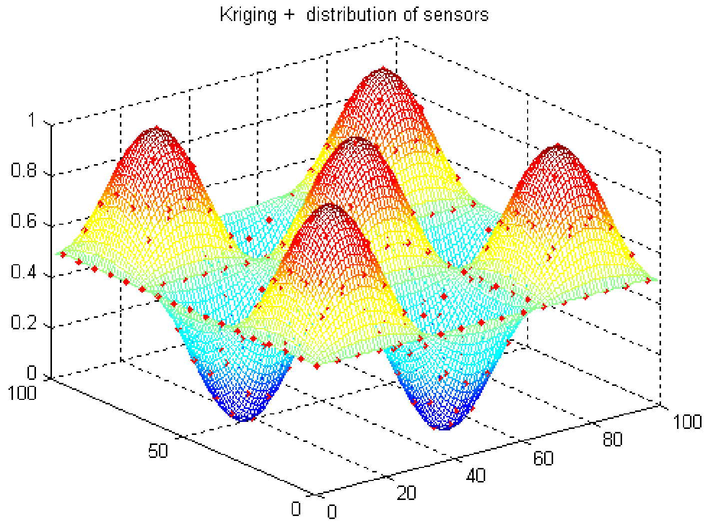
100*100

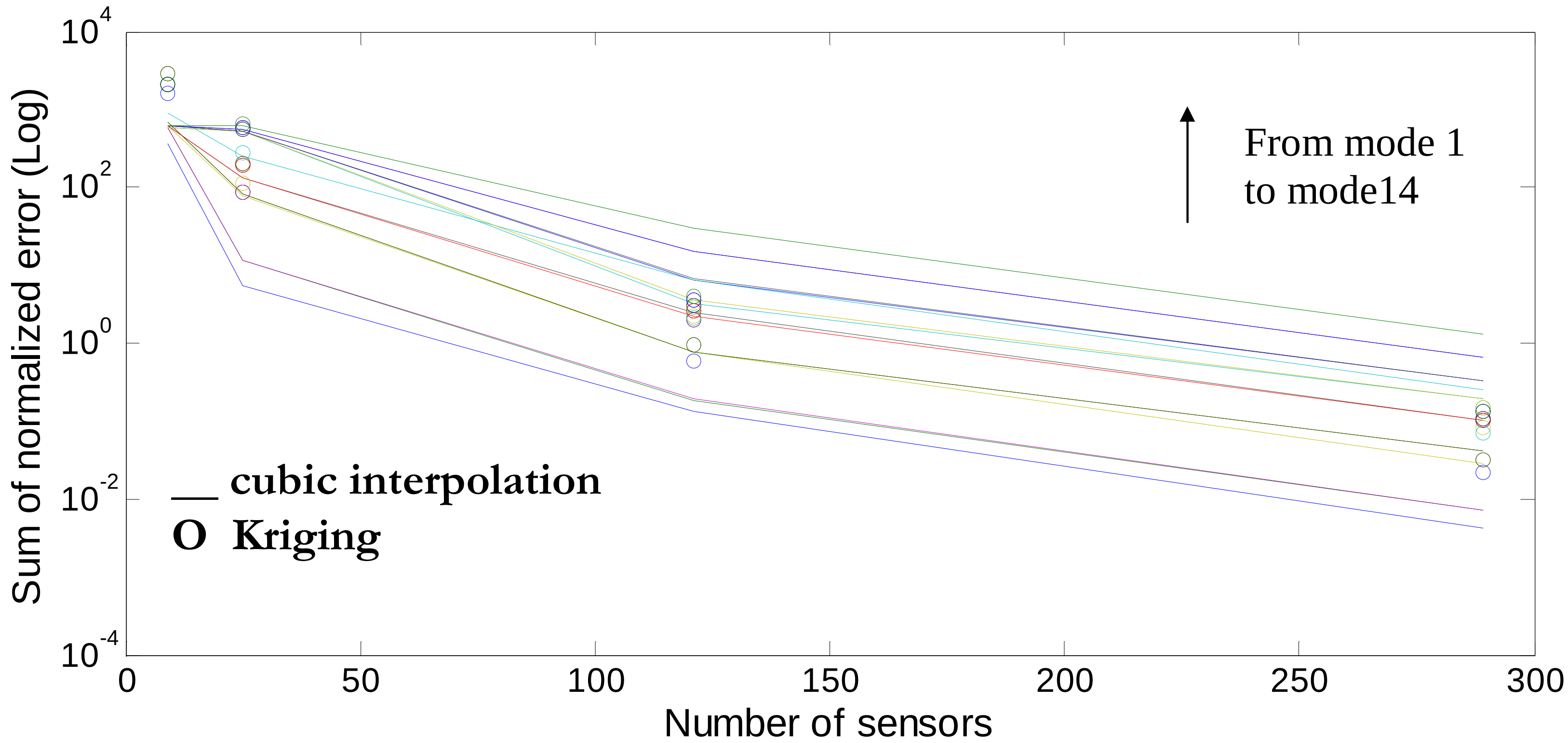






Reliability indicator: Sum of errors computed for several sensors density

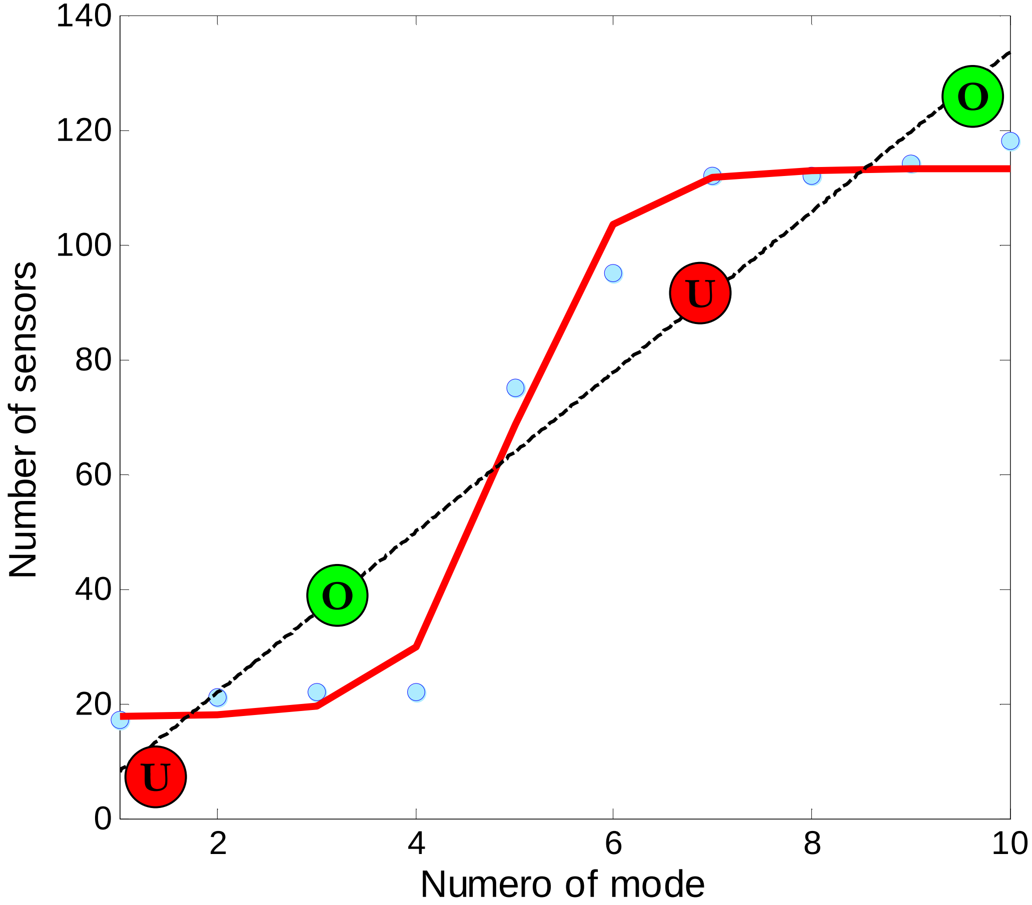




Decrease of the sum of normalized error of both method when number of sensors increase (the more the modeshape is complex the greater is the error).

Kriging method shows globally more stable results for important sensors density.

Regular grid — Optimal sensors density??



SSSS plate example:
10 modes in 0-700Hz

For correlation R between theoretical and reconstructed mode shape ($R^2>0.9$) we fit the sigmoid function

$$Y = (\max - \min)/(1 + \exp(-a - bX)) + \min =$$
$$(95.35)/(1 + \exp(4.93 - x)) + 17.73$$

O Oversampling

U Undersampling

$$y = 14x - 5.9$$

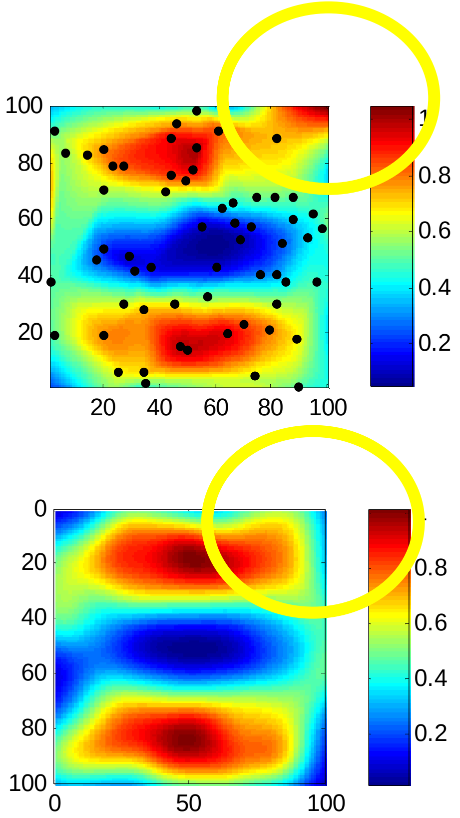
Sigmoid function fit (red line) of mode correctly identified with cubic interpolation with R=0.95 (blue circle) function of number of sensors needed to obtain this level of correlation

Optimal sensors density function of modeshape complexity ? Difficult to conclude even on this simple exemple…

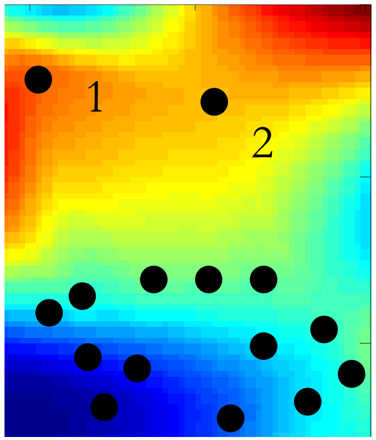
Random grid

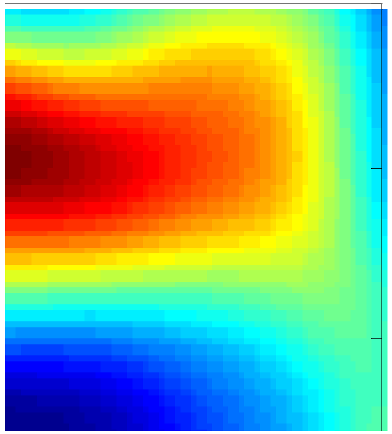
Using a priori information for SHM algorithm?

For SHM purpose it could be interesting to increase localy the density of sensors where the probability of defect is known to be important.



cubic interpolation





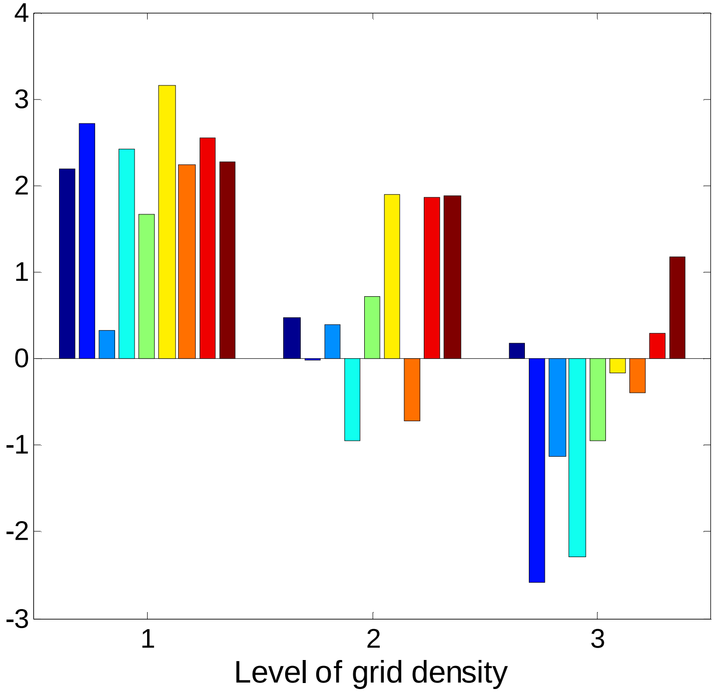
Kriging uses
observations of its value at nearby locations

kriging offers better estimation for the up right zone where only 2 sensors exist.

Random grid

Why using Monte Carlo approach?

$$DCK = log(MAE_{Kriging}) - log(MAE_{Cubic})$$



Levels of grid density are 10*10 (1), 20*20 (2), 30*30 (3).

As interpolation accuracy is strongly dependant of sensor placement (effect less important for regular grid),

we will use Monte Carlo simulation to make a statistical measure of the accuracy of random mapping method.

Using 100 tests of random grid interpolation between cubic and Kriging method for the 9 first modes.

In 71% of the cases, Kriging method highlight better estimation of the mode shape.

Kriging is globally more precise for randomized grid.

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Benchmark for damping estimation

Conclusions

Previous works

Previous works have demonstrated the ability of RFP method to distinguish resonance even at high noise level **[26]** among these methods:

The Complex-Exponential Method
The Hilbert-Envelope Method
The Ibrahim Time Domain method

The CWT method highlights better results in some minor cases: lower damping ratio corrupted with noise, separate modes **[27]**

Which method do we use for a SHM purpose ?

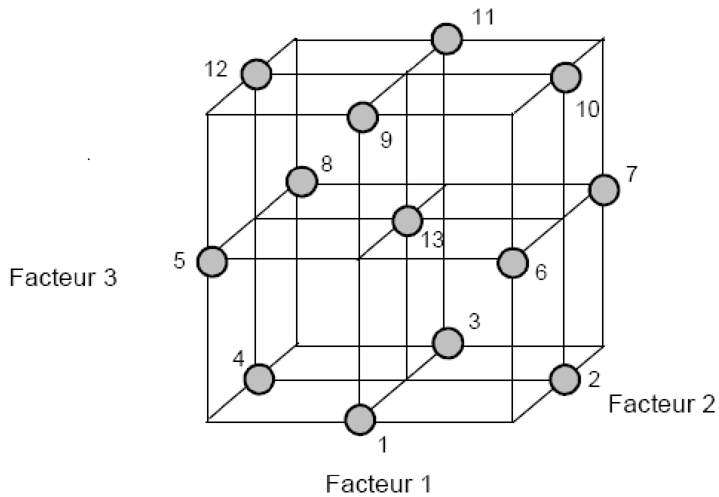
Solution: a more detailed supervised Benchmark !

[26] Iglesias A.M., Investigating Various Modal Analysis – Extracting Techniques to Estimate Damping Ratio, Master of Engineering, Virginia Polytechnic Institute, 2000.
[27] Wu J.Y., Extracting Damping Ratio Using Wavelets, Master of Engineering, Massachusetts Institute of Technology, 2001.

Design of experiments (DOE)

influence of key parameters on damping estimation

A study of sensitivity is presented in order to deduce the significant factors.
This study is carried out on simulated FRF with the aim of identifying the factors which have the most significant effect on the three damping estimation algorithms.



Frequency Resolution FR,
Signal to Noise Ratio SNR,
Damping Ratio Level DRL

$$P\_RFP = +0.3 + 0.4FR + 1.5SNR + 0.5DRL$$

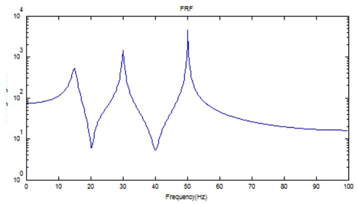
INTERACTIONS

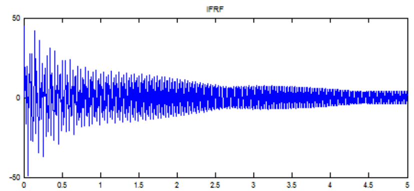
$$+ 0.7FR.SNR - 0FR.DRL + 0.8SNR.DRL + 0FR^2 + 1.2SNR^2 + 0DRL^2$$

The full factorial DOE requires 81 experiments as Box-Behnken design reduces the computational time into 27 experiments and the quality of the information you get will be higher.

Numerical benchmark

Analyse the accuracy and robustness of the 3 damping estimation algorithms





Step1: Signals simulation (IRF, FRF) of known properties

Frequency Resolution FR, Signal to Noise Ratio SNR, Damping Ratio Level DRL

Step2: Damping estimation using the 3 algorithms, computation of the errors

Step3: Compute the sensitivity of the estimation function of these parameters

Step4: Repeat the previous step with influence of the last parameter **Modal Density MD**

Step5: Compare the results (importance of the MD in the damping estimation)

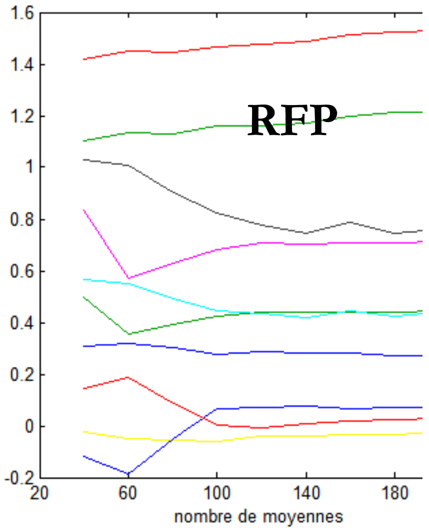
First DOE

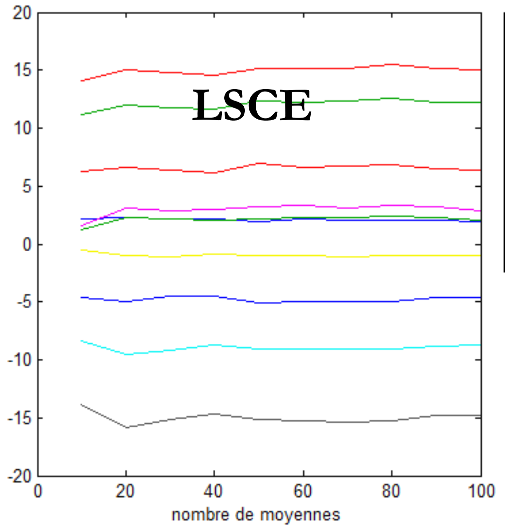
Level	***Low***	***Middle***	***High***
Signal to Noise Ratio (*SNR*)	80dB	40dB	20dB
Frequency resolution (*FR*)	0.2Hz	0.5Hz	1Hz
Damping Ratio Level (*DRL*)	0.1%	0.7%	4%
Modal Density (*MD*)	3 modes/35Hz	3 modes/15Hz	3 modes/7Hz

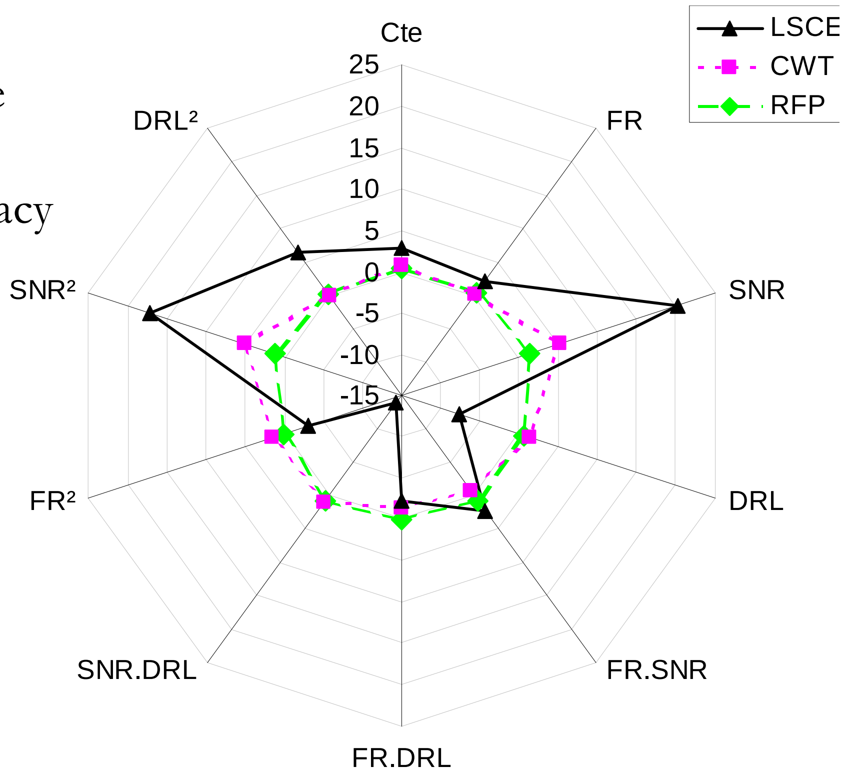
2nd DOE

Probabilistic identification | Monte Carlo approach

we have synthesised one noisy FRF by experiment and the white noise is recomputed at each iteration of the Monte Carlo simulation (100 steps).
This mixed approach permits to study not only the accuracy of the identification algorithm but also the robustness



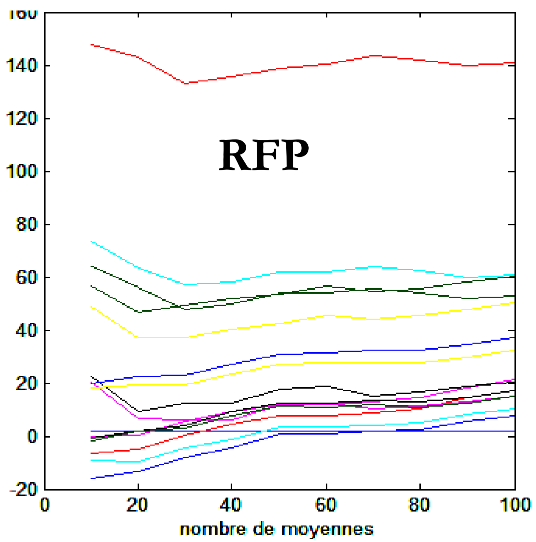


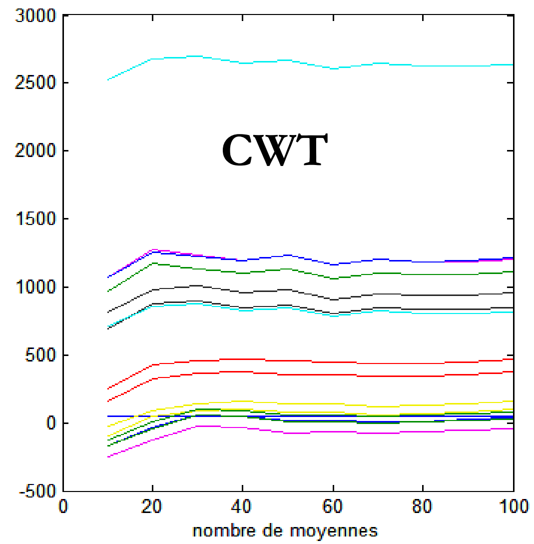


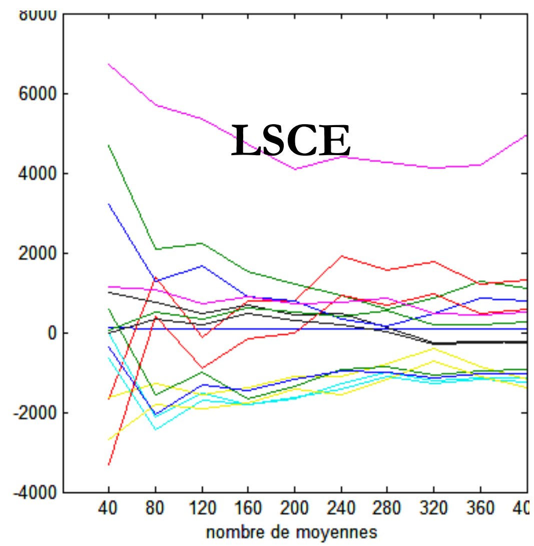
After 100 iterations the coefficients converge (the weight of the effects are stable).

Convergence of the 2nd DOE

Main influent parameter / method







	RFP	CWT	LSCE		RFP	CWT	LSCE
Cte	2	50,9	84,5	*FR . SNR*	43,8	58,7	-1171,7
FR	54,3	1097	1112,2	*FR . MD*	16,4	834,9	-279,2
SNR	140,4	430,1	1333,1	*FR . DRL*	29	1193,2	797,1
MD	62,3	2634,9	-1155,7	*SNR . MD*	53,9	44,4	-940,1
DRL	10,5	1194,5	4970,3	*SNR . DRL*	5,8	335	587

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Modeshape estimation

Surface reconstruction is a crucial point in modeshape changes based SHM method.
Random grid could be used in part of structure where there is a high probability of damage.
Kriging is better at high resolution: difficulty to use in classical modal analysis
Solution Optical Continuous Monitoring (High Speed High Resolution Camera)?

Damping Estimation

RFP More robust of the 3 methods with main sensibility to SNR
CWT has High sensibility to noise and also modal density parameter
LSCE Less accurate, high sensibility to all parameters

Conclusions

In a Structural Health Monitoring process, we are tracking a pole shifts (which induce modal parameters changes) **> We should have a robust damping estimation method but also a robust baseline model for comparison.**

1. Damping is difficult to estimate with accuracy, choice of the method crucial ?
2. Influent factors are dependent of the method
3. RFP is more accurate/robust (more physical ?)

> Future Works

Development of a mixed modeshape and damping damage detection tool on composites structures

Automatic tool to adapt the damping estimation algorithm to the SHM process