

Simplified modelling for the simulation of bolted and/or bonded joints.

Macro-element technique.

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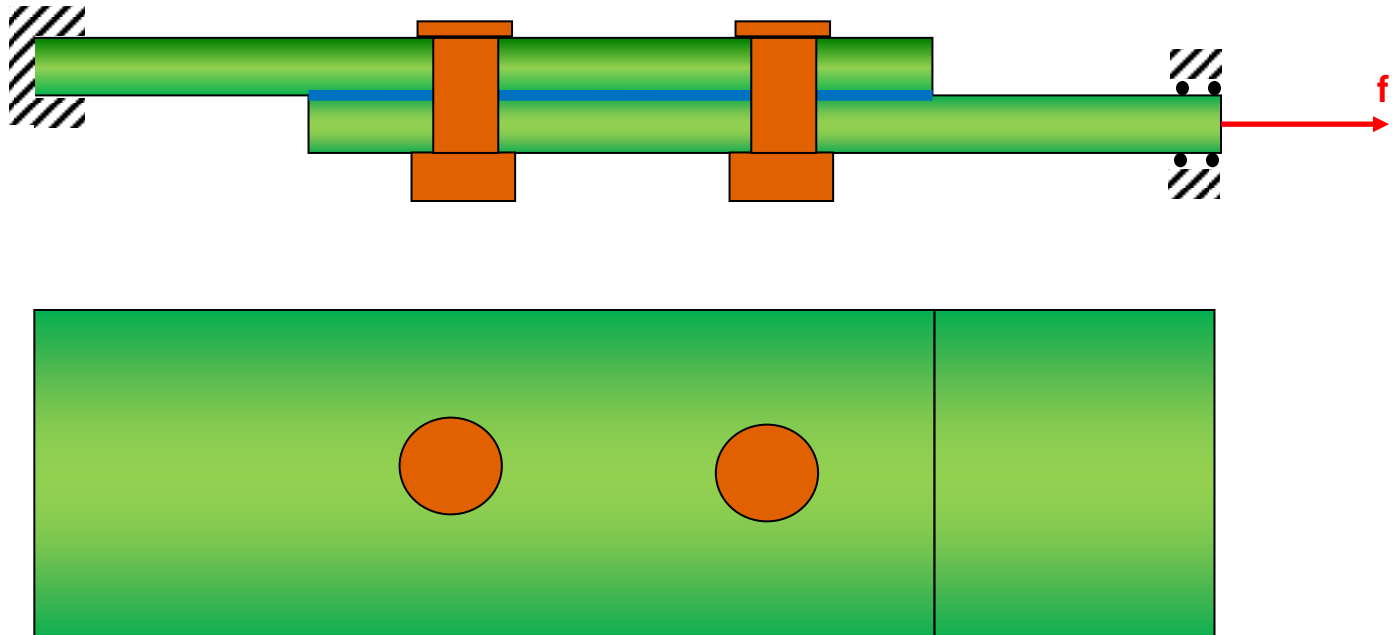


The work presented is the result of numerous contributions from several co-workers, among them:

- Pr. Marc Sartor (INSA Toulouse)
- Pr. Jacques Huet (ISAE-SUPAERO)
- Pr. Frédéric Lachaud (ISAE-SUPAERO)
- Sébastien Schwartz (Sogeti High Tech)
- Anthony Da Veiga (Sogeti High Tech)
- Dr. Guillaume Lélías (Sogeti High Tech, ISAE-SUPAERO)
- Pr. Joseph Morlier (ISAE-SUPAERO)

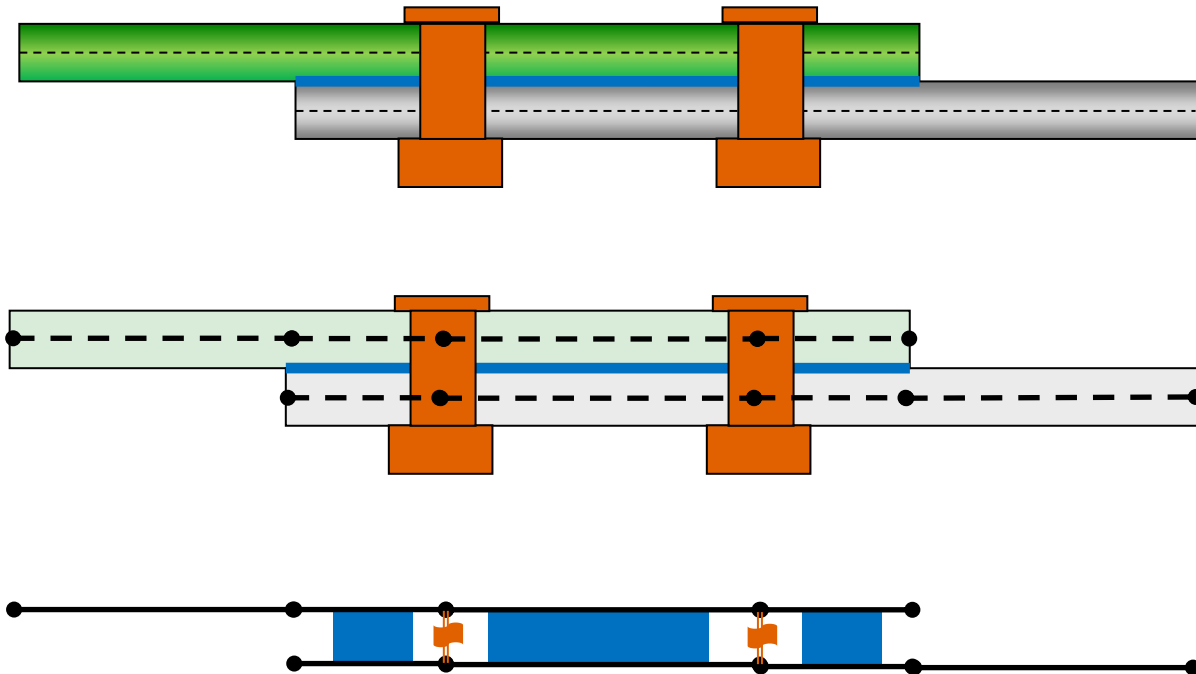
ORIGIN

Which **simplified model** to **quickly and accurately** assess the load transfer within a **hybrid (bolted / bonded) joint** in-plane loaded?



ORIGIN




Pr. Marc Sartor (INSA Toulouse) suggested to model the joints with special elements, termed macro-elements (**ME**). (Paroissien, 2006) (Paroissien et al., 2007a) (Paroissien et al., 2007b)



ORIGIN

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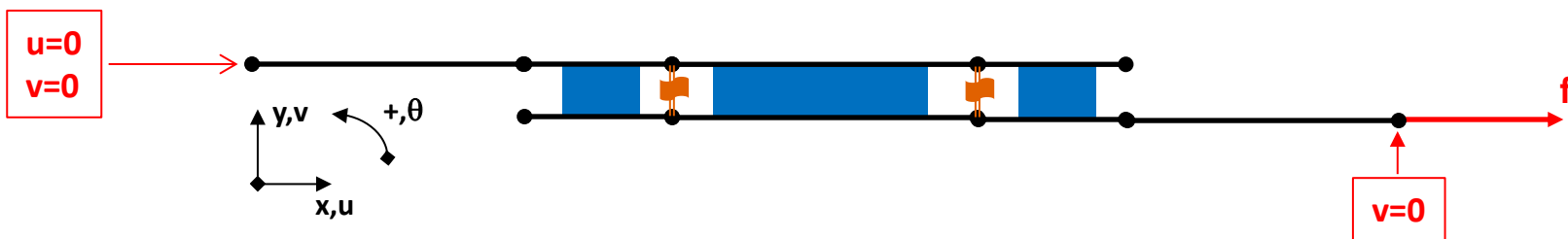
kinematics			
1D-bar	bar element	bonded-bars element (BBa element)	2 DoF fastener element
1D-beam	beam element	bonded-beams element (BBe element)	6 DoF fastener element
elementary stiffness matrix	K_{barre} K_{poutre}	K_{BBa} K_{BBe}	K_{F_3} K_{F_6}

ORIGIN

The methodology consists in:

1. to assemble the structural stiffness matrix from the elementary stiffness matrix K_s
2. to apply the boundary conditions
3. to minimize the potential energy leading to the linear system $F_s = K_s U_s$

The main difficulty is then the formulation of elementary stiffness matrices.



FASTENER MACRO-ELEMENT

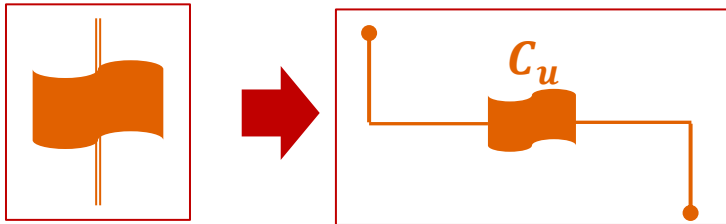
ELEMENTARY STIFFNESS MATRIX

The elementary stiffness matrix of fastener macro-element depends on the chosen kinematics.

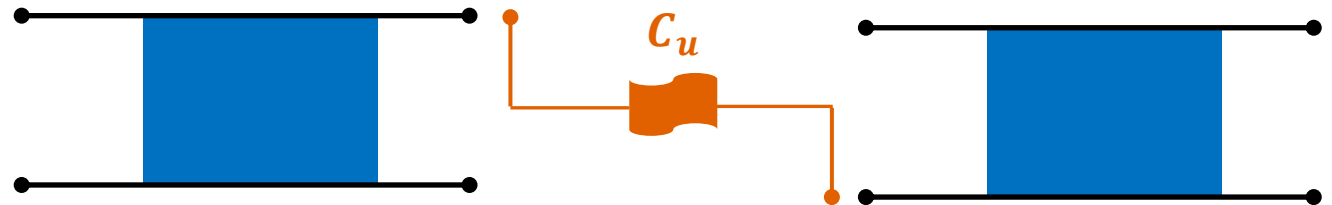
1D-bar

Shear spring

This modelling corresponds to the classical one used for the simplified stress analysis of bolted joint under 1D-bar kinematics (Ross, 1947).



$$K_{F-2} = C_u \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

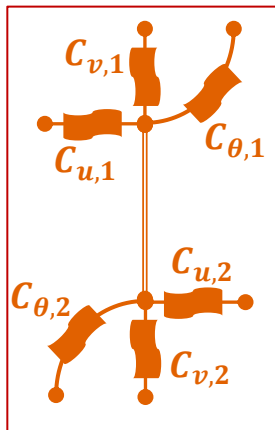
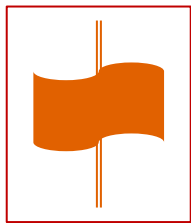


ELEMENTARY STIFFNESS MATRIX

The elementary stiffness matrix of fastener macro-element depends on the chosen kinematics.

1D- beam

6 springs + 1 RBE

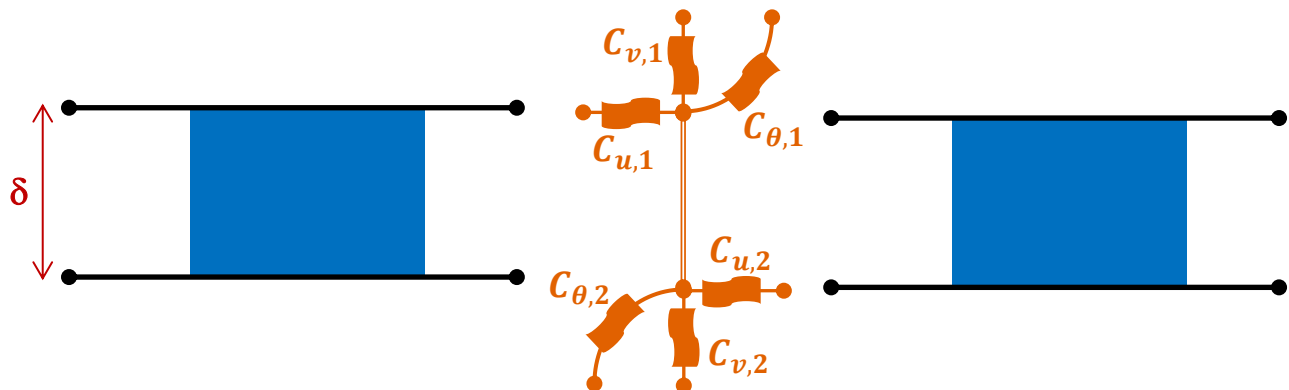


symmetrical fastener

$$K_{F-6} = \frac{1}{k} \begin{pmatrix} 2C_u C_\theta & -2C_u C_\theta & 0 & 0 & -\delta C_u C_\theta & -\delta C_u C_\theta \\ -2C_u C_\theta & 2C_u C_\theta & 0 & 0 & \delta C_u C_\theta & \delta C_u C_\theta \\ 0 & 0 & k C_v & -k C_v & 0 & 0 \\ 0 & 0 & -k C_v & k C_v & 0 & 0 \\ -\delta C_u C_\theta & \delta C_u C_\theta & 0 & 0 & 2C_\theta^2 + \delta^2 C_u C_\theta & -2C_\theta^2 \\ -\delta C_u C_\theta & \delta C_u C_\theta & 0 & 0 & 2C_\theta^2 & 2C_\theta^2 + \delta^2 C_u C_\theta \end{pmatrix}$$

$$k = 2C_\theta + \frac{\delta^2}{2} C_u$$

(Paroissien et al., 2017)



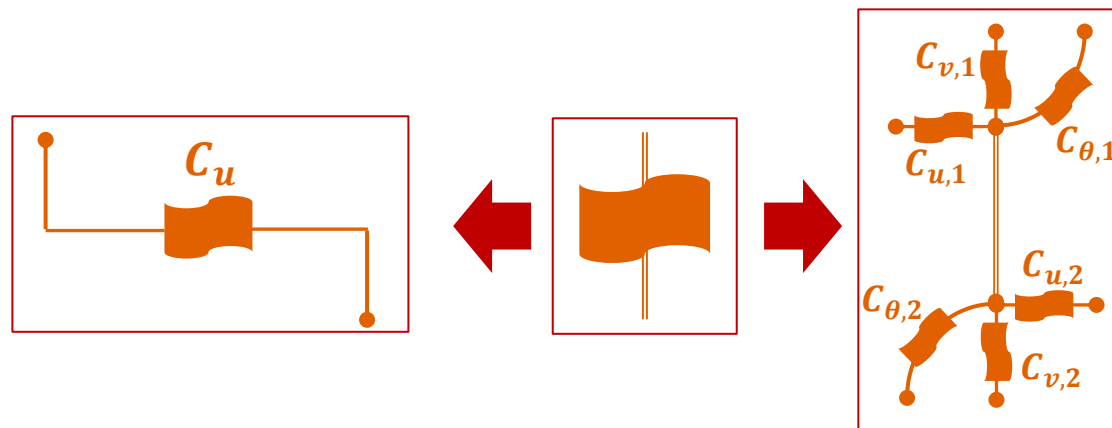
ELEMENTARY STIFFNESS MATRIX

The elementary stiffness matrices of fastener macro-element depend on fastener stiffness C_u , C_v and C_θ .

In the literature, it exists a large number of formulae to compute these stiffnesses such as:

(Tate and Rosenfeld, 1946) (Swift, 1984) (Huth, 1986) (Cope and Lacy, 2000) (Morris, 2004)

These fastener stiffnesses are regarded as global parameters representing for several local phenomenon. They can be assessed from experimental and numerical tests, in a tailored application field.



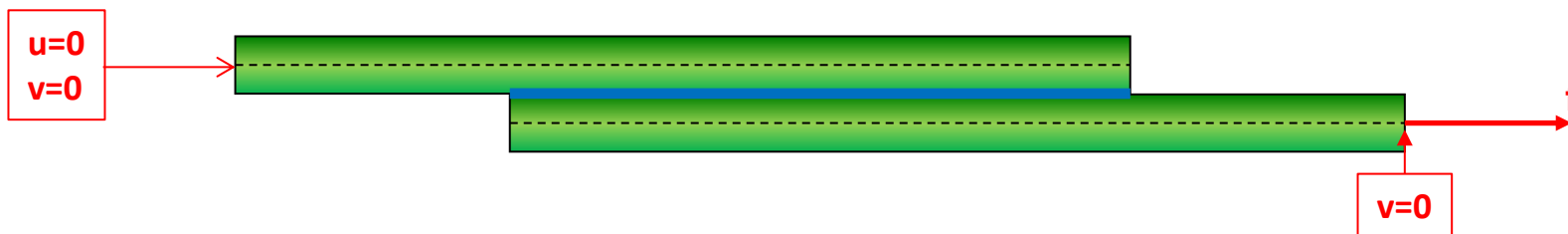
ELEMENTARY STIFFNESS MATRIX

A – Preamble

In the literature, it exists a large numbers of closed-form formulae to accurately predict the adhesive stress distribution along the overlap, such as detailed in: (van Ingen and Volt, 1993) (Tsai and Morton, 1994) (da Silva et al. 2009).

➤ it means that the choice of simplified hypotheses on which are based these formulae is judicious.

Nevertheless, the application field of these formulae is in general quite restricted. For example, Goland and Reissner provided in 1944 (Goland and Reissner, 1944) the adhesive shear and peel stress distribution for a balanced, simply supported, in plane loaded lap joint. But, there is not any more closed-form solution if the adherends are dissimilar or if the joint is clamped....



ELEMENTARY STIFFNESS MATRIX

A – Preamble

The judicious simplified hypotheses lead to a system of ODEs composed of the **constitutive equations** and of the **local equilibrium equations**.

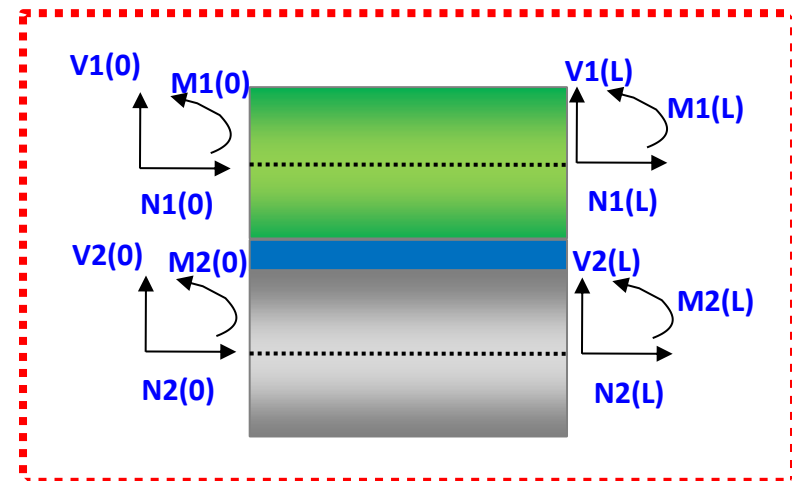
This system can be difficult to be solved in particular :

- when the **adherends** are **dissimilar**
- when the **boundary conditions** are given
- when the material are **non linear**

Goland and Reissner introduced a methodology for the stress analysis of bonded joints referred to **sandwich-type analysis**. It consists in analyzing the bonded overlap as function of internal loads and/or displacements joint extremities.

The adhesive layer is seen as **bed of springs** linked to the adherends **surface modelled as plates**.

Moreover, in the solution by Goland and Reissner takes into account for the **non linear geometrical effect** induced by the lag of the **neutral line**.



ELEMENTARY STIFFNESS MATRIX

A – Preamble

As a result, following Goland and Reissner, several other analyses have been published **to enlarge the application field**. Most of them used the sandwich type analysis, such as: (Hart-Smith 1973b) (Williams 1975) (Bigwood and Crocombe 1989) (Oplinger 1991) (Tsai et al. 1998) (Högberg 2004) (Nemes and Lachaud 2009) (Luo et Tong 2009) (Weißgraeber et al. 2014).

Other simplified stress analyses of bonded joints were published by modelling the adherends and the adhesive layer such as **2D continuum media** (Renton and Vinson, 1977) (Allman, 1977) (Adams and Mallick, 1992). This type of analysis allows for the compliance of the **free stress state** at overlap ends.

Nevertheless, in order to solve the system of ODEs in a large application field, **particular resolution schemes** have to be used such as:

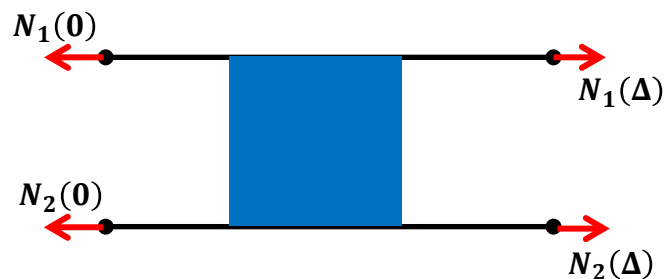
- « multisegment integration method » (Mortensen, 1997)
- the **macro-element technique** (Gustafson et al., 2006) (Paroissien, 2006)

BONDED-LAP MACRO-ELEMENT

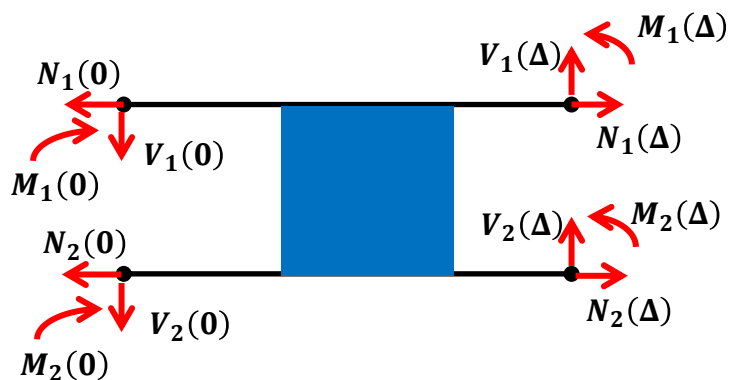
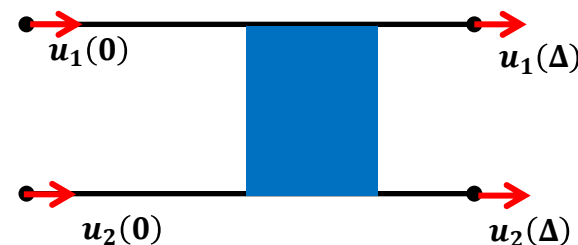
ELEMENTARY STIFFNESS MATRIX

B – Principle

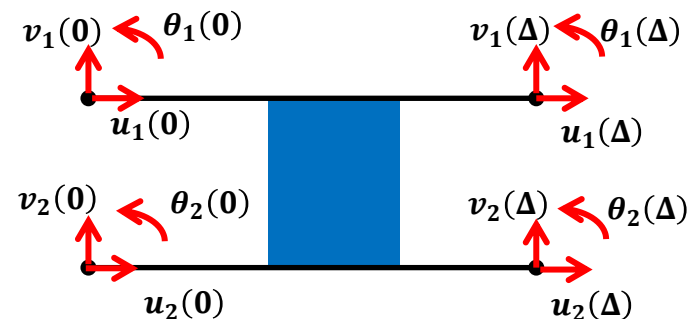
The elementary stiffness matrix for a bonded overlap expresses the relationships between the nodal displacements and the nodal forces.



$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \end{pmatrix} = K_{BC} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \end{pmatrix}$$



$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \\ -V_1(0) \\ -V_2(0) \\ V_1(\Delta) \\ V_2(\Delta) \\ -M_1(0) \\ -M_2(0) \\ M_1(\Delta) \\ M_2(\Delta) \end{pmatrix} = K_{PC} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \\ v_1(0) \\ v_2(0) \\ v_1(\Delta) \\ v_2(\Delta) \\ \theta_1(0) \\ \theta_2(0) \\ \theta_1(\Delta) \\ \theta_2(\Delta) \end{pmatrix}$$

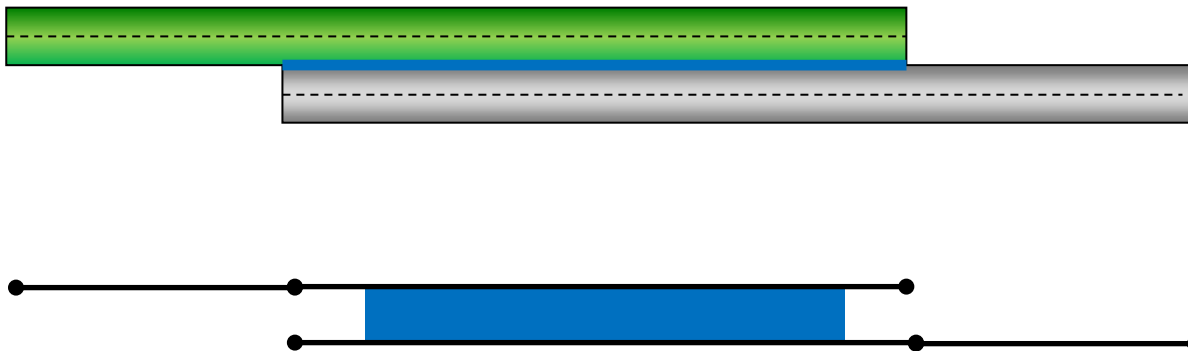


ELEMENTARY STIFFNESS MATRIX

B – Principle

Contrary to the classical FE, **the shape of interpolation functions is not assumed** a priori. The shape of interpolation functions has the shape of functions solving the ODEs.

One significant consequence is that only one ME is needed to model an entire bonded overlap (linear elastic analysis). The displacements, internal forces and adhesive stresses are obtained at any abscissa of the overlap.



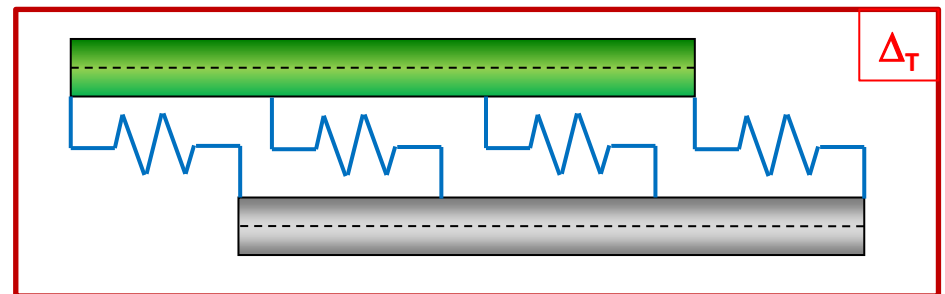
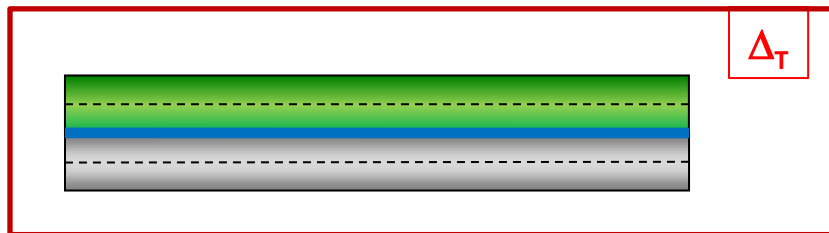
- 6 nodes
- 6 dof in 1D-bar
- 18 dof in 1D-beam

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

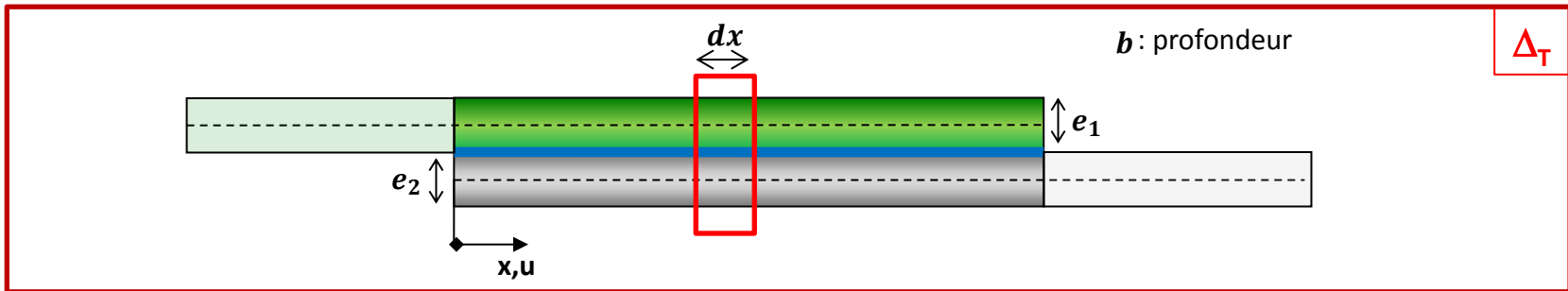
The hypotheses are:

- homogeneous linear elastic material behavior
- local equilibrium of Volkersen ([Volkersen, 1938](#))
- the adherends are modelled as bars, with eventually a linear variation of the shear stress with the thickness ([Tsaï et al., 1998](#))
- the adhesive layer is modelled as a bed of shear springs
- the adhesive thickness e_a is constant
- mechanical loading and application of a uniform temperature variation Δ_T

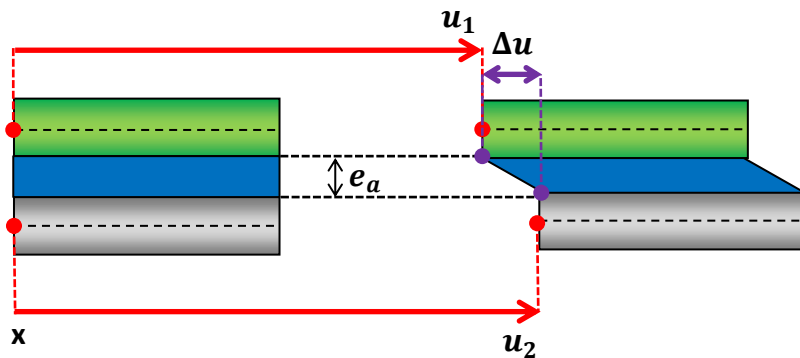


ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

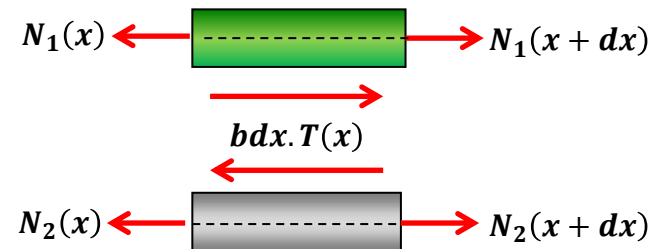


constitutive equations



$$\frac{du_j}{dx} = \frac{N_j}{A_j} + \alpha_j \Delta_T \quad A_j = E_j b e_j \quad T = \frac{G_a}{e_a} \Delta u$$

local equilibrium of Volkersen



$$\frac{dN_1}{dx} = -bT \quad \frac{dN_2}{dx} = bT$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

From the constitutive equations and the local equilibrium equations, a system of ODE's in the adherend longitudinal displacements can be written and solved.

$$\frac{dN_j}{dx} = (-1)^j bT, j = 1,2$$

$$T = \frac{G_a}{e_a} (u_2 - u_1)$$

$$\frac{du_j}{dx} = \frac{N_j}{A_j} + \alpha_j \Delta T, j = 1,2$$



$$\frac{d^2 u_1}{dx^2} + \frac{G_a}{e_a} \frac{1}{e_1 E_1} (u_2 - u_1) = 0$$

$$\frac{d^2 u_2}{dx^2} - \frac{G_a}{e_a} \frac{1}{e_2 E_2} (u_2 - u_1) = 0$$



$$u_1 = \frac{1}{2} [c_1 + c_2 x - c_3(1 + \chi)e^{-\eta x} - c_4(1 + \chi)e^{\eta x}]$$

$$u_2 = \frac{1}{2} [c_1 + c_2 x + c_3(1 - \chi)e^{-\eta x} + c_4(1 - \chi)e^{\eta x}]$$

$$\eta^2 = \frac{G_a}{e_a} \left(\frac{1}{e_1 E_1} + \frac{1}{e_2 E_2} \right) \quad \chi = \frac{\frac{1}{e_1 E_1} - \frac{1}{e_2 E_2}}{\frac{1}{e_1 E_1} + \frac{1}{e_2 E_2}}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

The 4 integration constants c_i can then be expressed as functions of the 4 nodal displacements.

$$\begin{cases} u_1 = \frac{1}{2} [c_1 + c_2 x - c_3(1 + \chi)e^{-\eta x} - c_4(1 + \chi)e^{\eta x}] \\ u_2 = \frac{1}{2} [c_1 + c_2 x + c_3(1 - \chi)e^{-\eta x} + c_4(1 - \chi)e^{\eta x}] \end{cases} \Rightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = M_e^{-1} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \end{pmatrix} \Leftrightarrow \mathbf{C} = M_e^{-1} \mathbf{U}_e$$

$$M_e^{-1} = \begin{pmatrix} \frac{(1 - \chi)}{(1 - \chi)} \frac{\Delta}{e^{\eta \Delta}} & \frac{(1 + \chi)}{(1 + \chi)} \frac{\Delta}{e^{\eta \Delta}} & \frac{0}{\Delta} & \frac{0}{\Delta} \\ -\frac{2 \sinh \eta \Delta}{e^{-\eta \Delta}} & \frac{2 \sinh \eta \Delta}{e^{\eta \Delta}} & \frac{1}{2 \sinh \eta \Delta} & -\frac{1}{2 \sinh \eta \Delta} \\ \frac{2 \sinh \eta \Delta}{e^{-\eta \Delta}} & -\frac{2 \sinh \eta \Delta}{e^{\eta \Delta}} & \frac{1}{2 \sinh \eta \Delta} & \frac{1}{2 \sinh \eta \Delta} \end{pmatrix}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

From the constitutive equation of adherends, the normal forces can be deduced as functions of integration constants:

$$\frac{N_1}{A_1} = \frac{du_1}{dx} - \alpha_1 \Delta T$$

$$\frac{N_2}{A_2} = \frac{du_2}{dx} - \alpha_2 \Delta T$$



$$N_1(x) = \frac{1}{2} [c_2 + c_3 \eta (1 + \chi) e^{-\eta x} - \eta c_4 (1 + \chi) e^{\eta x}] A_1 - A_1 \alpha_1 \Delta T$$

$$N_2(x) = \frac{1}{2} [c_2 - c_3 \eta (1 - \chi) e^{-\eta x} + \eta c_4 (1 - \chi) e^{\eta x}] A_2 - A_2 \alpha_2 \Delta T$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

From the constitutive equation of adherends, the normal forces can be deduced as functions of integration constants:

$$\left\{ \begin{array}{l} \frac{N_1}{A_1} = \frac{du_1}{dx} - \alpha_1 \Delta T \\ \frac{N_2}{A_2} = \frac{du_2}{dx} - \alpha_2 \Delta T \end{array} \right. \rightarrow \left\{ \begin{array}{l} N_1(x) = \frac{1}{2} [c_2 + c_3 \eta (1 + \chi) e^{-\eta x} - \eta c_4 (1 + \chi) e^{\eta x}] A_1 - A_1 \alpha_1 \Delta T \\ N_2(x) = \frac{1}{2} [c_2 - c_3 \eta (1 - \chi) e^{-\eta x} + \eta c_4 (1 - \chi) e^{\eta x}] A_2 - A_2 \alpha_2 \Delta T \end{array} \right.$$

The nodal forces are then expressed as functions of integration constants such as:

$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \end{pmatrix} + \begin{pmatrix} -A_1 \alpha_1 \\ -A_2 \alpha_2 \\ A_1 \alpha_1 \\ A_2 \alpha_2 \end{pmatrix} \Delta T = N_e \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \Leftrightarrow \mathbf{F}_e + \mathbf{F}_{therm} = N_e \mathbf{C}$$

$$N_e = \frac{1}{2} \begin{pmatrix} 0 & -A_1 & -\eta(1 + \chi)A_1 & \eta(1 + \chi)A_1 \\ 0 & -A_2 & \eta(1 - \chi)A_2 & -\eta(1 - \chi)A_2 \\ 0 & A_1 & \eta(1 + \chi)e^{-\eta\Delta}A_1 & -\eta(1 + \chi)e^{\eta\Delta}A_1 \\ 0 & A_2 & -\eta(1 - \chi)e^{-\eta\Delta}A_2 & \eta(1 - \chi)e^{\eta\Delta}A_2 \end{pmatrix}$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-bar

For the 1D-bar kinematics, it is then possible to obtain the expressions for the components of the elementary stiffness matrix K_{BBa} (Paroissien, 2006) (Paroissien, 2007a).

$$C = M_e^{-1} U_e$$

$$F_e + F_{therm} = N_e C$$



$$F_e + F_{therm} = \boxed{N_e M_e^{-1} U_e}$$

K_{BBa}

$$K_{BBa} = \frac{1}{1 + \chi_A} \frac{A_2}{\Delta} \begin{pmatrix} \frac{\eta\Delta}{\tanh \eta\Delta} + \frac{1}{\chi_A} & 1 - \frac{\eta\Delta}{\tanh \eta\Delta} & -\frac{\eta\Delta}{\sinh \eta\Delta} - \frac{1}{\chi_A} & \frac{\eta\Delta}{\sinh \eta\Delta} - 1 \\ 1 - \frac{\eta\Delta}{\tanh \eta\Delta} & \frac{\eta\Delta}{\tanh \eta\Delta} + \chi_A & \frac{\eta\Delta}{\sinh \eta\Delta} - 1 & -\frac{\eta\Delta}{\sinh \eta\Delta} - \chi_A \\ -\frac{\eta\Delta}{\sinh \eta\Delta} - \frac{1}{\chi_A} & \frac{\eta\Delta}{\sinh \eta\Delta} - 1 & \frac{\eta\Delta}{\tanh \eta\Delta} + \frac{1}{\chi_A} & 1 - \frac{\eta\Delta}{\tanh \eta\Delta} \\ \frac{\eta\Delta}{\sinh \eta\Delta} - 1 & -\frac{\eta\Delta}{\sinh \eta\Delta} - \chi_A & 1 - \frac{\eta\Delta}{\tanh \eta\Delta} & \frac{\eta\Delta}{\tanh \eta\Delta} + \chi_A \end{pmatrix}$$

$$\chi_A = \frac{A_2}{A_1}$$

SOLUTION

C – 1D-bar

The resolution of $F_S = K_S U_S$ allows for the determination of the nodal displacement vector: U_S

For each ME, the vector of nodal displacement is then known: U_e

Thus, for each ME, the integration constants are obtained such as: $C = M_e^{-1} U_e$

As result, the internal loads, the displacements and adhesive shear stress are obtained at any x:

$$u_1 = \frac{1}{2} [c_1 + c_2 x - c_3 (1 + \chi) e^{-\eta x} - c_4 (1 + \chi) e^{\eta x}]$$

$$u_2 = \frac{1}{2} [c_1 + c_2 x + c_3 (1 - \chi) e^{-\eta x} + c_4 (1 - \chi) e^{\eta x}]$$

$$T = \frac{G_a}{e_a} (u_2 - u_1)$$

$$N_1(x) = \frac{1}{2} [c_2 + c_3 \eta (1 + \chi) e^{-\eta x} - \eta c_4 (1 + \chi) e^{\eta x}] A_1 - A_1 \alpha_1 \Delta T$$

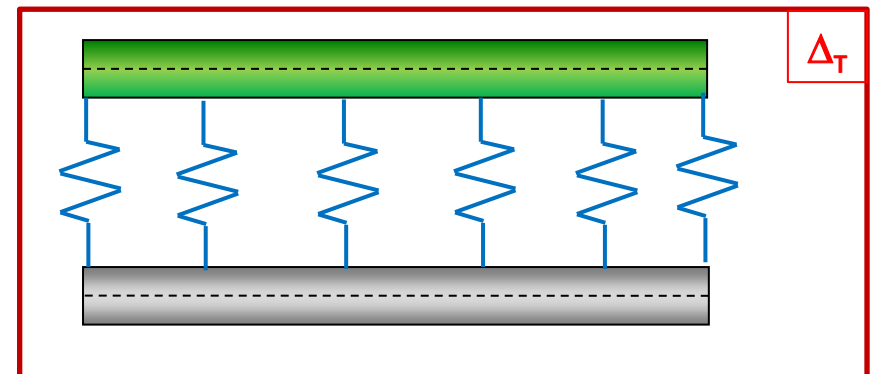
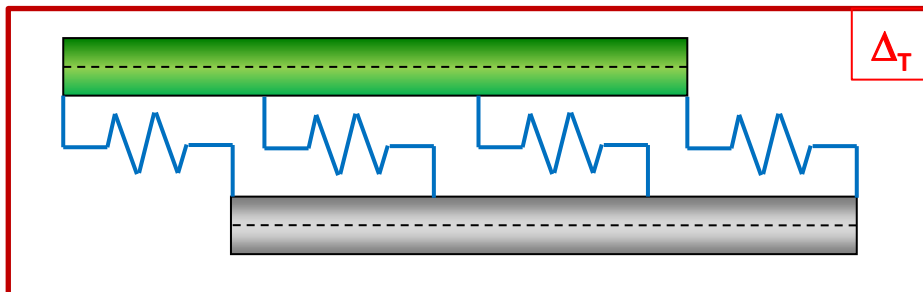
$$N_2(x) = \frac{1}{2} [c_2 - c_3 \eta (1 - \chi) e^{-\eta x} + \eta c_4 (1 - \chi) e^{\eta x}] A_2 - A_2 \alpha_2 \Delta T$$

ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

The hypotheses are:

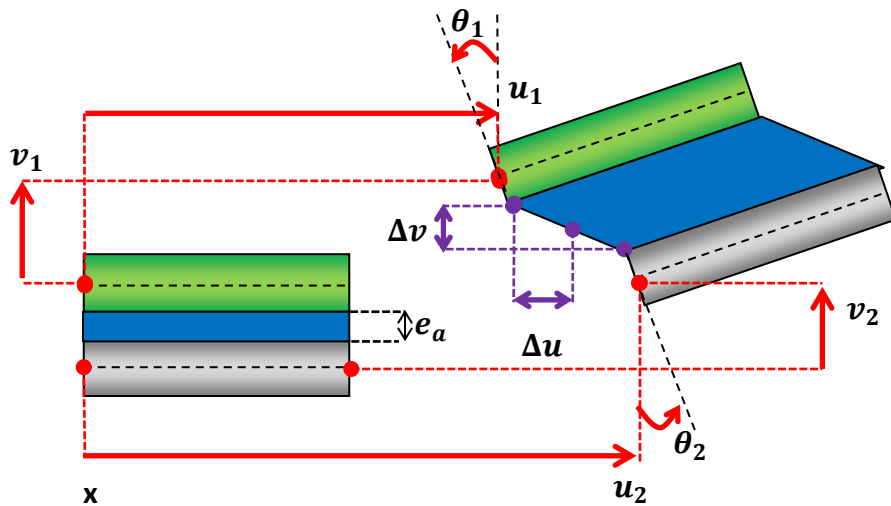
- linear elastic material behavior
- local equilibrium of Goland and Reissner ([Goland and Reissner, 1944](#))
- the adherends are modelled as laminated Euler-Bernoulli beams, with eventually a linear variation of the shear stress with the thickness ([Tsai et al., 1998](#))
- the adhesive layer is modelled as a bed of shear and peel springs
- the adhesive thickness e_a is constant
- mechanical loading and application of a uniform temperature variation Δ_T



ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

constitutive equations

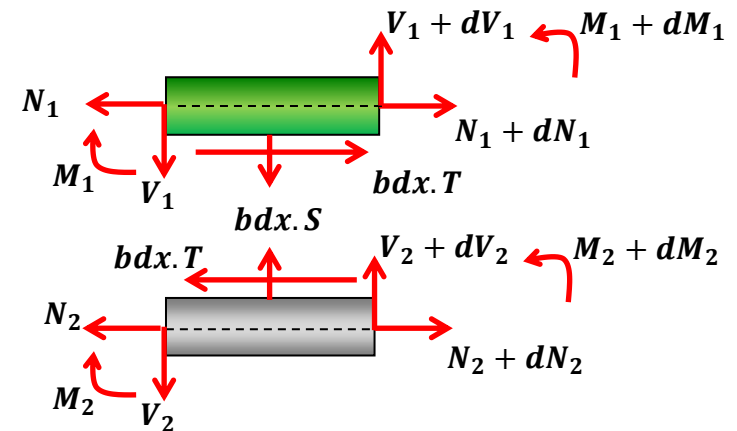


$$N_j = A_j \frac{du_j}{dx} - B_j \frac{d\theta_j}{dx} \quad M_j = -B_j \frac{du_j}{dx} + D_j \frac{d\theta_j}{dx}$$

$$\theta_j = \frac{dv_j}{dx}$$

$$T = \frac{G_a}{e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right) \quad S = \frac{E_a}{e_a} \Delta v$$

local equilibrium of Goland and Reissner



$$\frac{dN_j}{dx} = (-1)^j bT$$

$$\frac{dV_j}{dx} = (-1)^{j+1} bS$$

$$\frac{dM_j}{dx} + V_j + b \frac{e_j}{2} T = 0$$

BONDED-LAP MACRO-ELEMENT

ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

Contrary to the 1D-bar case, the closed-form expressions for the components of the elementary stiffness matrix are not obtained. However, the resolution scheme consisting in:

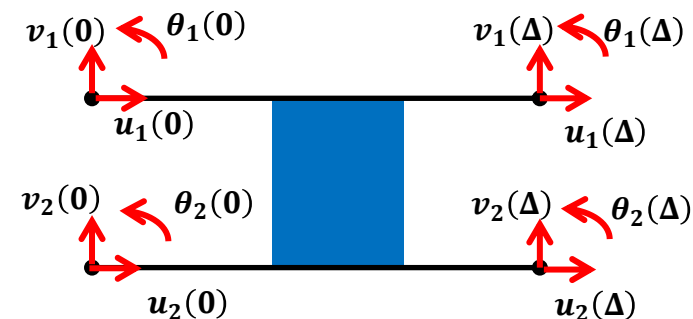
- determining the shape of solutions in displacements as functions of integration constants
- deducing the shape of internal loads as functions of integration constants

can be applied.

The difficulty here is the identification of a set of 12 free integration constants among the 27 appearing during the employed mathematical path.

$$\begin{pmatrix} -N_1(0) \\ -N_2(0) \\ N_1(\Delta) \\ N_2(\Delta) \\ -V_1(0) \\ -V_2(0) \\ V_1(\Delta) \\ V_2(\Delta) \\ -M_1(0) \\ -M_2(0) \\ M_1(\Delta) \\ M_2(\Delta) \end{pmatrix} + \begin{pmatrix} -N_1^{\Delta T} \\ -N_2^{\Delta T} \\ N_1^{\Delta T} \\ N_2^{\Delta T} \\ 0 \\ 0 \\ 0 \\ 0 \\ M_1^{\Delta T} \\ M_2^{\Delta T} \\ -M_1^{\Delta T} \\ -M_2^{\Delta T} \end{pmatrix} = K_{PC} \begin{pmatrix} u_1(0) \\ u_2(0) \\ u_1(\Delta) \\ u_2(\Delta) \\ v_1(0) \\ v_2(0) \\ v_1(\Delta) \\ v_2(\Delta) \\ \theta_1(0) \\ \theta_2(0) \\ \theta_1(\Delta) \\ \theta_2(\Delta) \end{pmatrix}$$

$$F_e + F_{therm} = N_e M_e^{-1} U_e = K_{PC} U_e$$



ELEMENTARY STIFFNESS MATRIX

C – 1D-beam

Another resolution scheme is developed since 2014. It is based on the resolution of the system of 1st order ODEs in the internal loads and displacements, making use of the exponential matrix (Paroissien et al., 2018a) (Paroissien et al., 2018b).

$$\frac{dN_j}{dx} = (-1)^j b \frac{G_a}{e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right)$$

$$\frac{dV_j}{dx} = (-1)^{j+1} b \frac{E_a}{e_a} (v_1 - v_2)$$

$$\frac{dM_j}{dx} = -V_j - b \frac{e_j G_a}{2 e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right)$$

$$\frac{du_j}{dx} = \frac{D_j N_j + B_j M_j}{\Delta_j}$$

$$\frac{dv_j}{dx} = \theta_j$$

$$\frac{d\theta_j}{dx} = \frac{A_j M_j + B_j N_j}{\Delta_j}$$



$$\frac{dX}{dx} = AX$$

$$X = \begin{pmatrix} N_1 \\ V_1 \\ M_1 \\ N_2 \\ V_2 \\ M_2 \\ u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{pmatrix}$$



$$\begin{cases} \Phi_A(X=0) = \expm(A \cdot 0) \\ \Phi_A(X=\Delta) = \expm(A \cdot \Delta) \end{cases}$$



$$M_e, N_e, K_{PC} = N_e M_e^{-1}$$

BONDED-LAP MACRO-ELEMENT

ELEMENTARY STIFFNESS MATRIX

C – 1D poutre

This resolution scheme allows for the fast formulation of the elementary stiffness matrix when the simplified hypotheses are modified. For example: Timoshenko beam, local equilibrium of Hart-Smith ([Hart-Smith, 1973](#)) or of Luo and Tong ([Luo et Tong, 2009](#)).

$$\frac{dN_j}{dx} = (-1)^j b \frac{G_a}{e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right)$$

$$\frac{dV_j}{dx} = (-1)^{j+1} b \frac{E_a}{e_a} (v_1 - v_2)$$

$$\frac{dM_j}{dx} = -V_j - b \frac{e_j G_a}{2 e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right)$$

$$\frac{du_j}{dx} = \frac{D_j N_j + B_j M_j}{\Delta_j}$$

$$\frac{dv_j}{dx} = \theta_j$$

$$\frac{dv_j}{dx} = \frac{V_j}{H_i} + \theta_j$$

$$\frac{d\theta_j}{dx} = \frac{A_j M_j + B_j N_j}{\Delta_j}$$

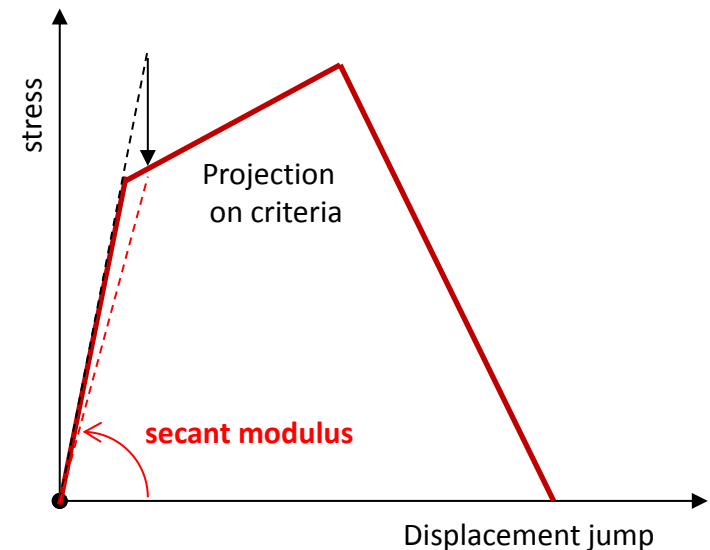
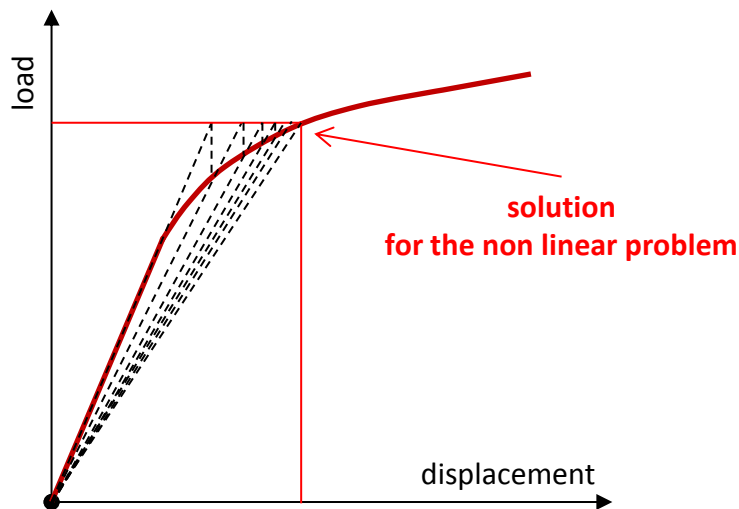
$$\frac{dM_j}{dx} = -V_j - b \frac{e_j G_a}{2 e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right) - \theta_j N_j$$

$$\frac{dM_j}{dx} = -V_j - b \frac{(e_j + e_a) G_a}{2 e_a} \left(u_2 - u_1 - \frac{e_2}{2} \theta_2 - \frac{e_1}{2} \theta_1 \right)$$

NONLINEAR COMPUTATION

The non linear material behavior of the adhesive layer and of fasteners can be taken into account thanks to computation scheme based on the Newton-Raphson iterative scheme using the secant stiffness matrix (Lélias et al., 2015).

In the case of non linear adhesive material, a mesh along the overlap is then required in order to be able to update the elementary stiffness matrix of each ME.

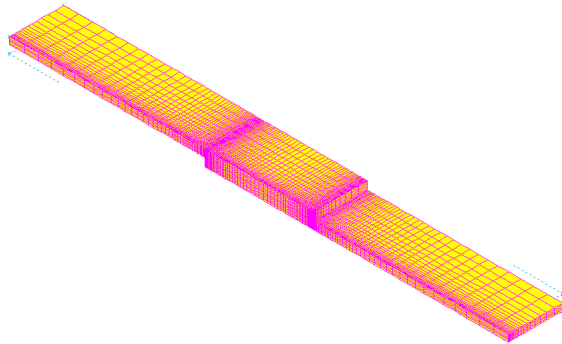


Various behavior laws are available (elasto-plastic, damaging evolution (CZM)) or will be available (visco-elastic, visco-plastic).

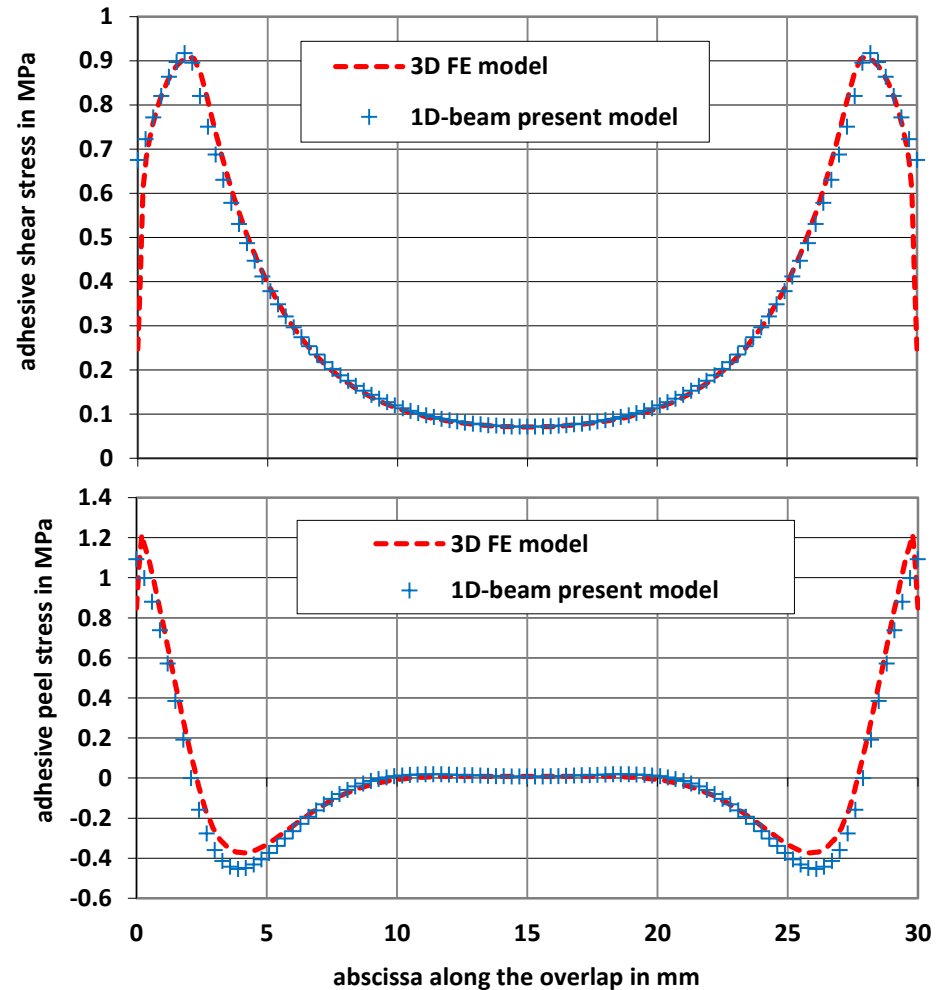
CPU TIME REDUCTION

Unbalanced single-lap bonded joint with an elastic perfectly plastic adhesive material in plane loaded
(Paroissien et al., 2013a).

The adhesive stresses are read on the **converged** 3D FE model on the neutral line of the adhesive layer.



Benefit in CPU time: x50



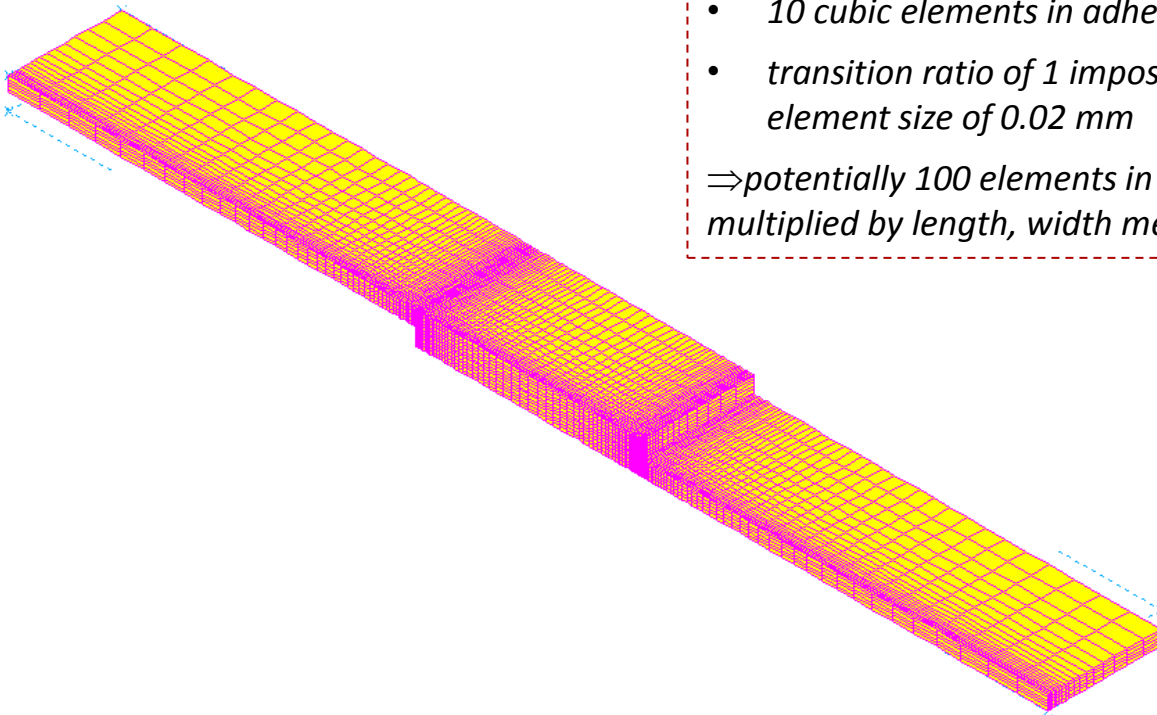
CPU TIME REDUCTION

The simulation of the mechanical behavior of bonded joints using the FEM is time consuming due to the relative difference in thickness between the adherends and the adhesive layers.

Example of single-lap bonded joint in 3D:

- *adherend thickness: 2 mm / adhesive thickness: 0.2 mm*
- *10 cubic elements in adhesive thickness = 0.02 mm each*
- *transition ratio of 1 imposed at the adhesive interface, an element size of 0.02 mm*

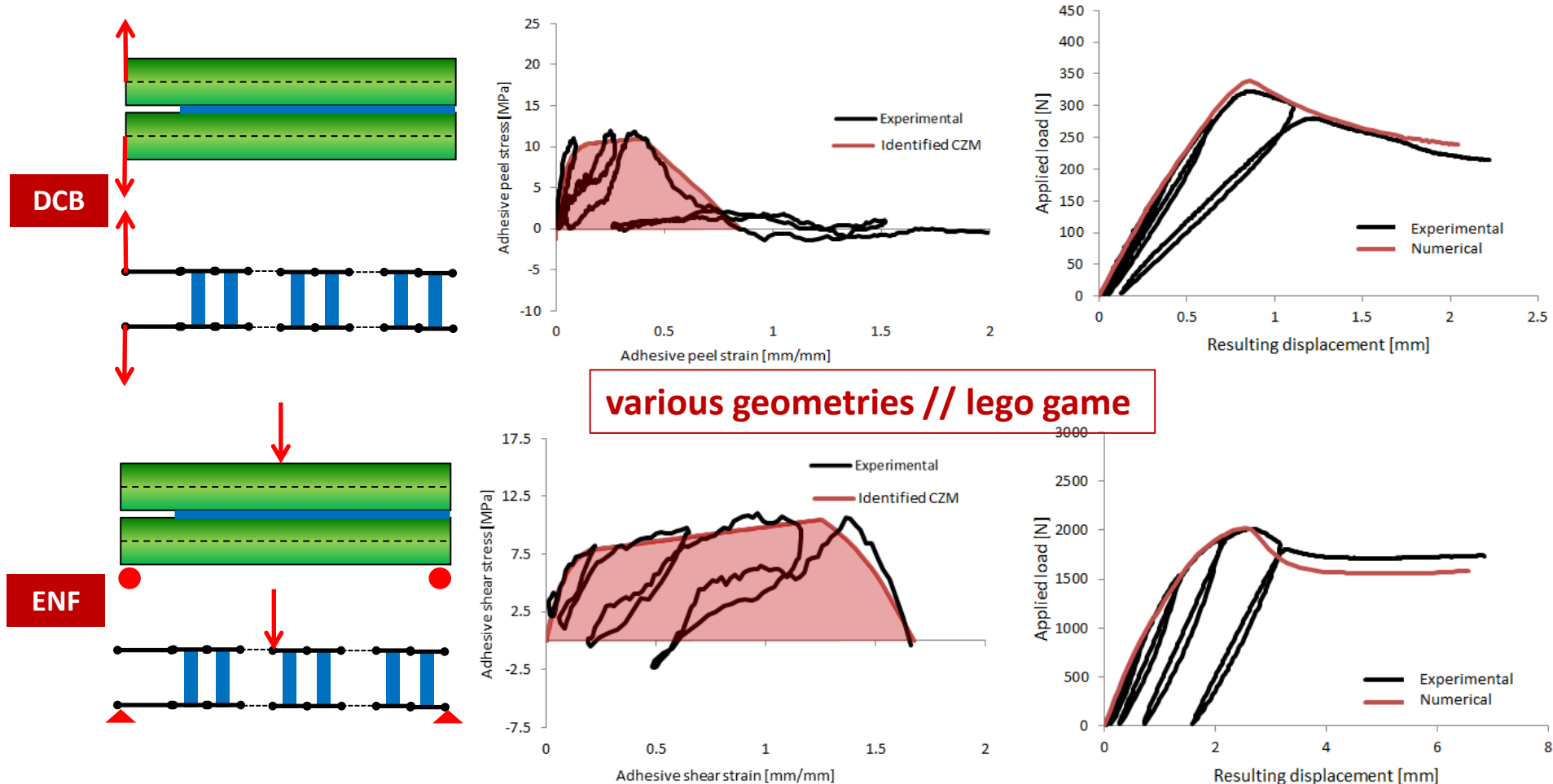
⇒ potentially 100 elements in the adherend thickness, to be multiplied by length, width mesh parameters...



APPLICATION CASES

ASSESSMENT OF CZM FOR THIN ADHESIVE LAYERS

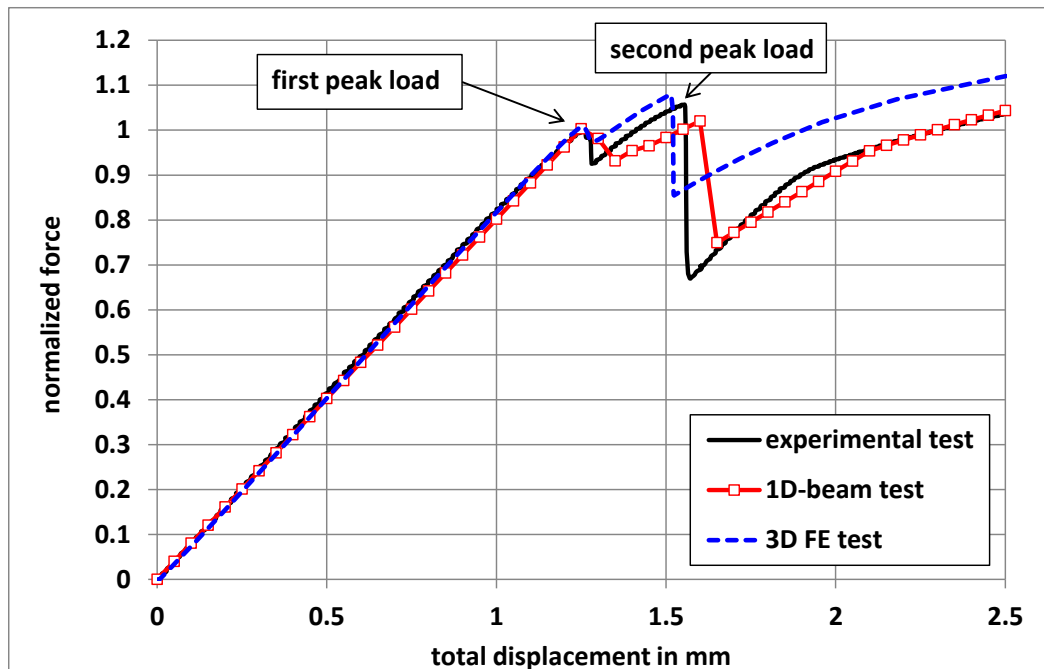
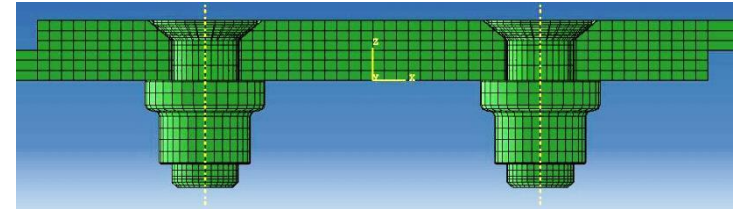
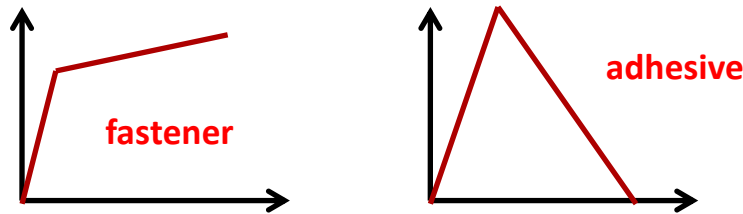
The ME technique has successively been used for the assessment of CZM for thin adhesive layers (Lélias, 2016) (Lélias et al., 2018).



APPLICATION CASES

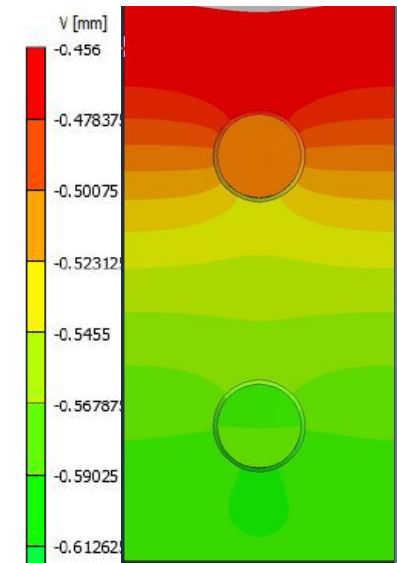
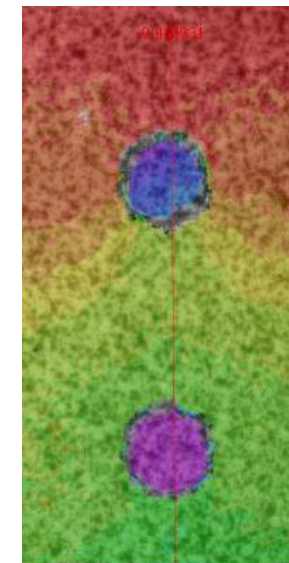
HYBRID (BOLTED/BONDED) JOINTS

Comparison between experimental test, 3D FE test and 1D-beam ME test of a single lap HBB joint in-plane loaded (Paroissien et al., 2017).



experimental

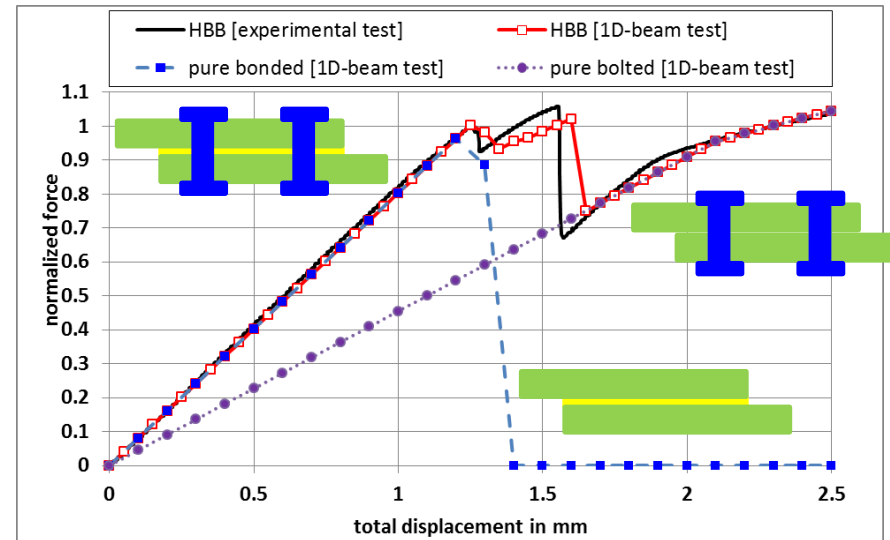
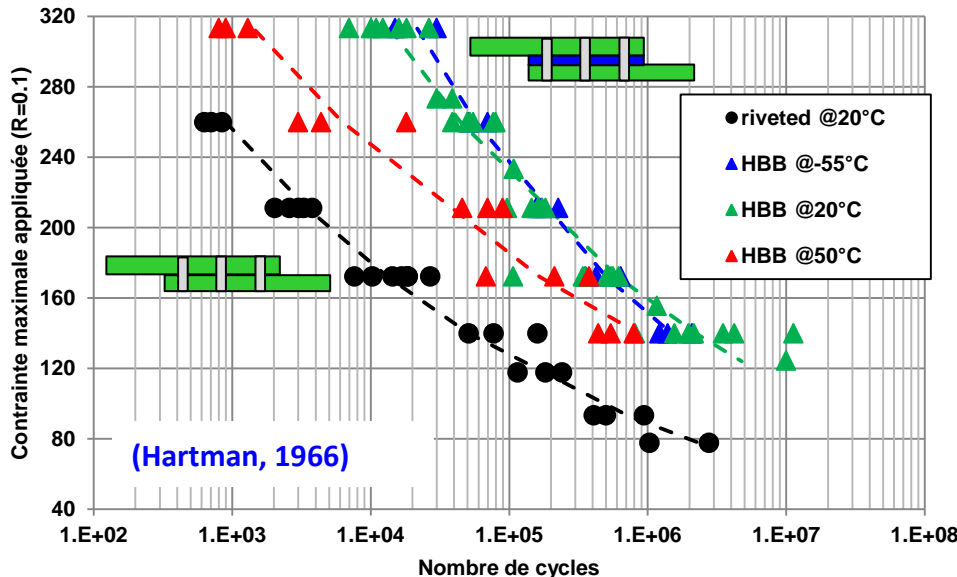
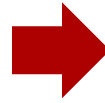
3D FE



APPLICATION CASES

HYBRID (BOLTED/BONDED)

Static and fatigue performance better than pure bolted or pure bonded, if the adhesive is judiciously chosen.



Fatigue strength prediction from ME output:

- for the holes with classical semi-empirical uni-axial approaches (Müller, 1995)
- for the adhesive layer through the progressive degradation of CZM (Khomarishad, 2011)

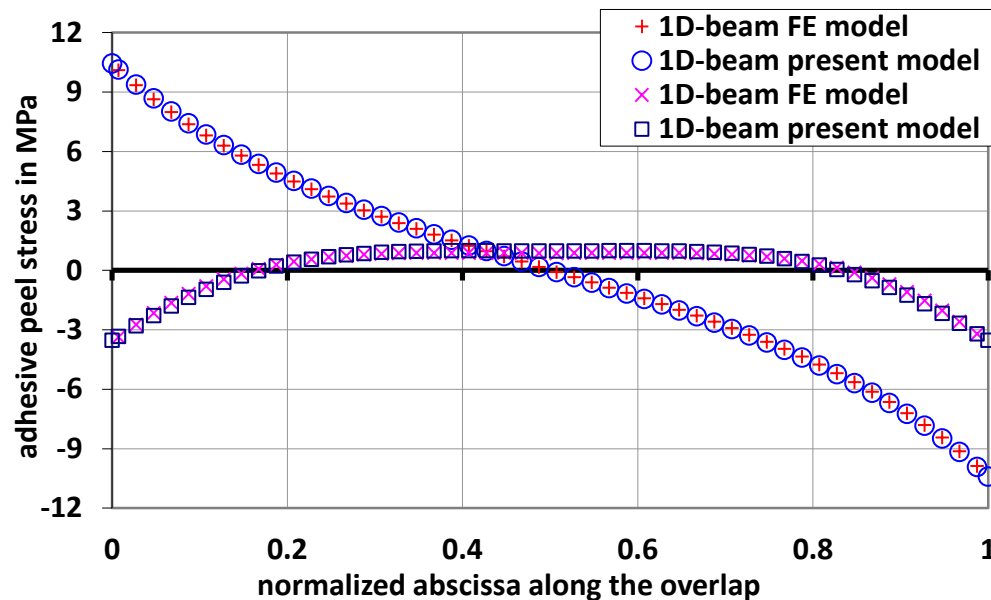
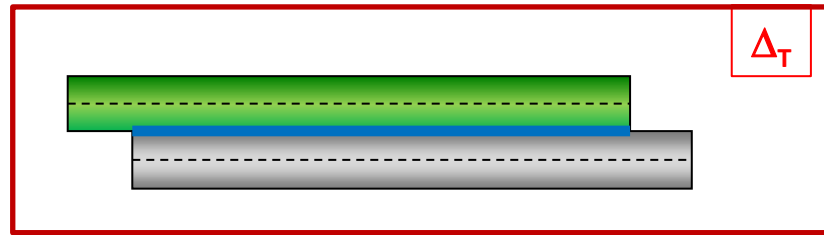


BONDED JOINTS UNDER THERMAL LOADING

Comparison between 1D FE model and 1D ME model (Paroissien et al., 2013b).

The 1D FE model is built with beam and spring elements.

Unbalanced single-lap joints under pure thermal loading under membrane and bending.



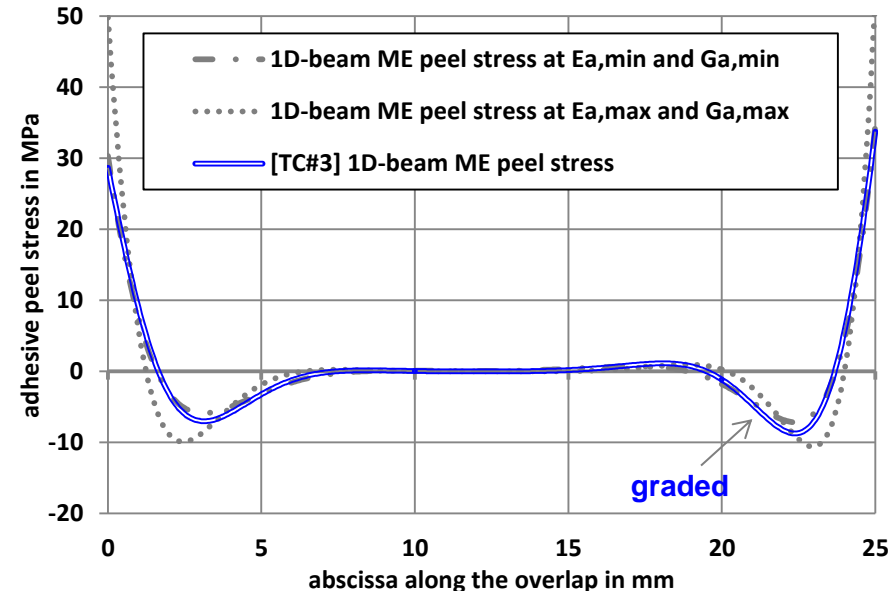
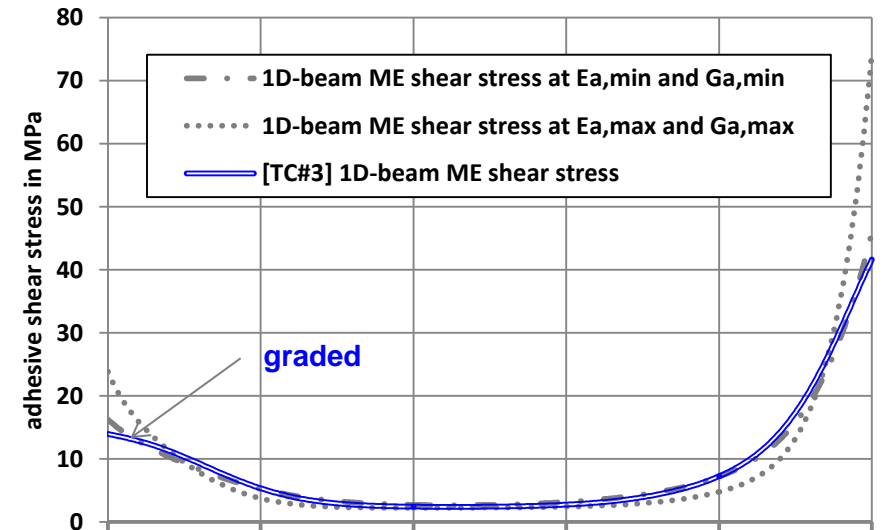
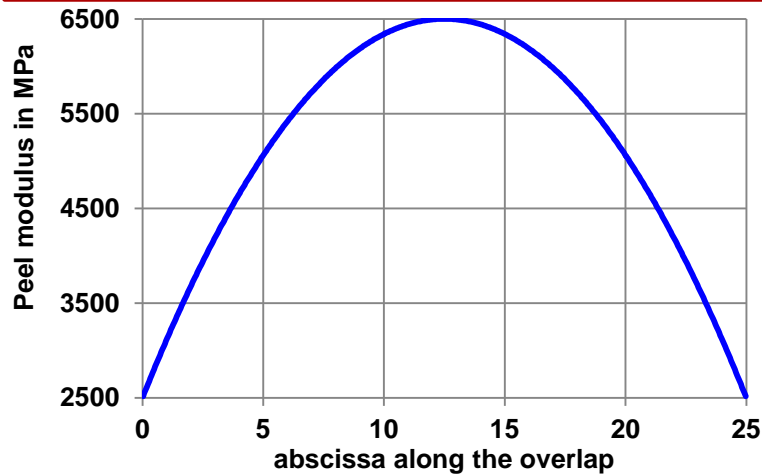
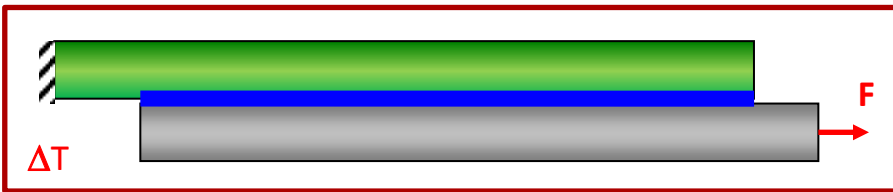
APPLICATION CASES

FUNCTIONNALLY GRADED ADHESIVE JOINTS

in collaboration with University of Porto

Unbalanced single-lap joints under combined mechanical and thermal loading with a symmetrical parabolic gradation (Paroissien et al., 2018b).

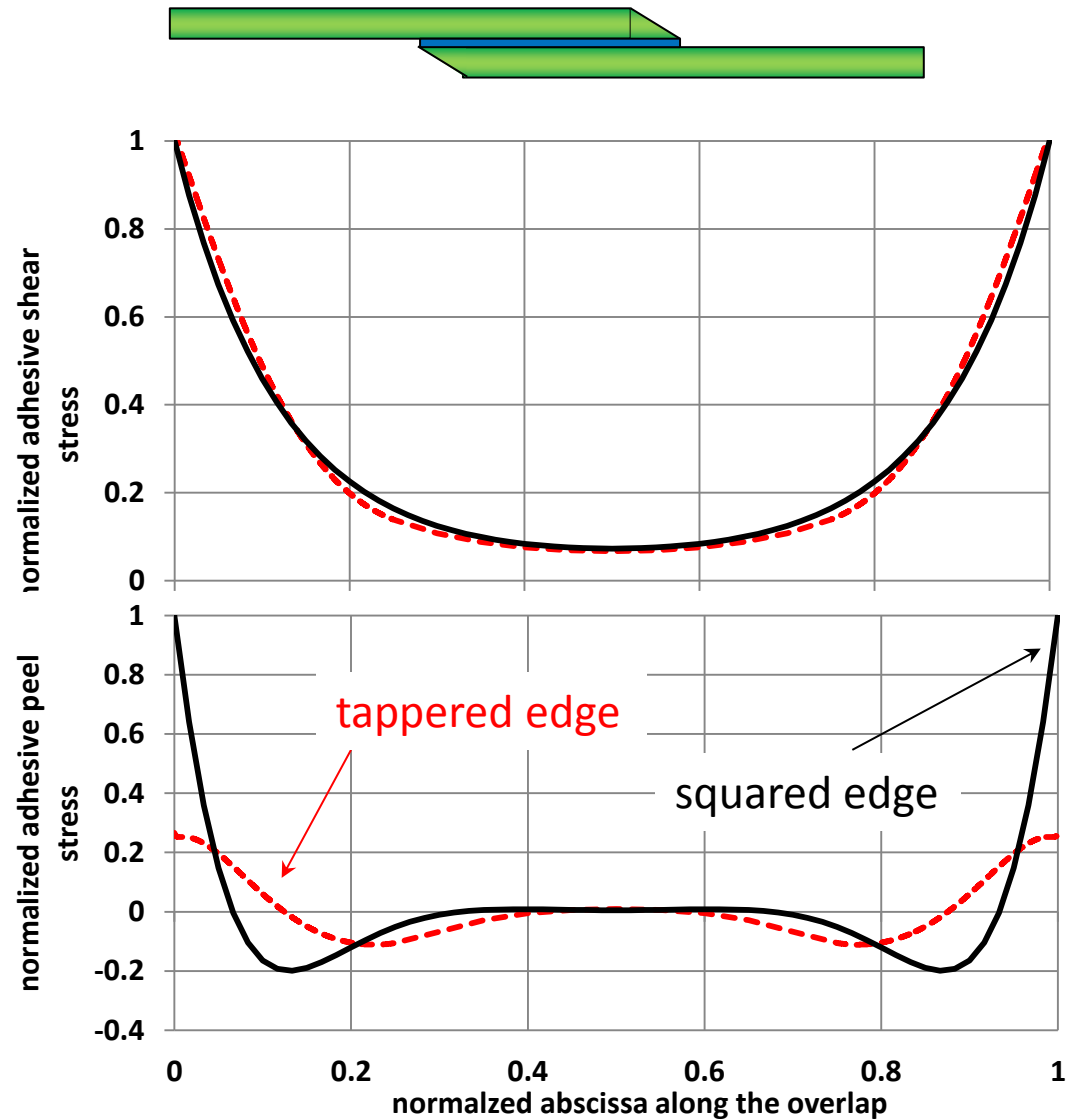
- reduction of peak stresses
- the simplified model offers a solution to optimize the gradation



REDUCTION OF PEAK STRESSES

single-lap joint with tapered adherend

- reduction of peak peel stresses



Prediction of failure at adherend/adhesive interface [PRACCOMET]

- PhD Thesis by Thiago Birro (2017-2020) supervised by ¹Frédéric Lachaud, ²Maëlen Aufray and ¹Éric Paroissien
- funded by Région Occitanie and ISAE-SUPAERO (APR2017 UFT MiP)
- Collaboration with CIRIMAT: **TACCOS**
- **experimental and numerical modelling of adherend/adhesive interface behavior**



Assessment of constitutive behavior of thin adhesive layer [S3PAC]

- PhD Thesis by Agathe Jaillon (2017-2020) supervised by ¹Frédéric Lachaud, ³Julien Jumel and ¹Éric Paroissien
- funded by BPI France, Région Occitanie and Région Nouvelle Aquitaine
- FUI (21) S3PAC
- **experimental and numerical modelling of cohesive behavior as function of adhesive thickness**



¹Institut Clément Ader (ICA), Université de Toulouse, ISAE-SUPAERO, INSA, IMT MINES ALBI, UTIII, CNRS, France

²CIRIMAT, Université de Toulouse, CNRS, INPT, UPS, France

³Université de Bordeaux, Arts et Metiers ParisTech, CNRS, I2M, UMR 5295, France

Macro-element of bonded plates [SCODyn]

- PhD Thesis by Benjamin Ordonneau (2018-2021) supervised by ¹Michel Salaün and ¹Éric Paroissien
- funded by CETIM and DGA
- **formulation of elementary stiffness and mass matrices of bonded overlap under 3D loading**

Dual functionalisation strength / fragmentation [SIMPACOS]

- PhD Thesis by Lorraine Silva (2018-2021) supervised by ¹Christine Espinosa and ⁴Lucas FM da Silva
- funded by ED MEGeP
- **particle-based numerical simulation to predict strength and controlled fragmentation for space structures**

¹Institut Clément Ader (ICA), Université de Toulouse, ISAE-SUPAERO, INSA, IMT MINES ALBI, UTIII, CNRS, France

⁴Department of Mechanical Engineering, Faculty of Engineering, University of Porto, Portugal

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