Ecole des JDMACS 2011 - GT MOSAR
Synthèse de correcteur: approches basées sur la forme estimation/commande

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Outline

Outline

Introduction.

The objective of this short course is to present 2 tools:

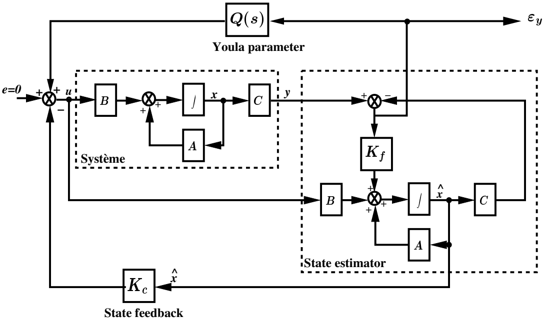
- the observer-based realization of a given controller: to implement high order controller (provided by optimization tools) giving a physical meaning to controller state variables,
- the cross standard form: a simple solution to the inverse optimal control problem. That is a canonical augmented standard plant whose unique H_∞ or H_2 optimal controller is a given controller. The **reverse engineering** idea is to apply the CSF to a given controller in order to set up a standard problem which can be completed to handle frequency-domain H_2 or H_∞ specifications.

Outline

Outline

Observer-based realization of a given controller - Principle.

(see D. Alazard and P. Apkarian in IJRNC'99)



Input:

plant : (A, B, C)

controller : $\begin{cases} \dot{x}_K = A_K x_K + B_K y \\ u = C_K x_K + D_K y \end{cases}$

x_K :??

Output: extented LQG realization

K_c , K_f , $Q(s)$

$\hat{x}\ /\ \dot{(x - \hat{x})} = (A - K_f C)(x - \hat{x})$

Observer-based ... - Principle

The observer-based state-space realization of the controller is the lower Linear Fractional Transformation of $Y(s)$ and $Q(s)$ where $Y(s)$ is defined by (non restrictive assumuption: plant direct feedthrough is null: $D = 0$):

$$\left[\begin{array}{c} \dot{\widehat{x}} \\ \hline u \\ \varepsilon_y \end{array}\right] = \left[\begin{array}{c|cc} A - BK_c - K_f C & K_f & B \\ \hline -K_c & 0 & I_m \\ -C & I_p & 0 \end{array}\right] \left[\begin{array}{c} \widehat{x} \\ \hline y \\ e \end{array}\right] . \qquad (1)$$

That is:

$$K(s) = F_l(Y(s), Q(s)) \,. \qquad (2)$$

Observer-based ... - Principle

From the observer-based realization, the separation principle reads:

- the closed-loop eigenvalues can be separated into n closed-loop state-feedback poles (spec$(A - BK_c)$), n closed-loop state-estimator poles (spec$(A - K_f C)$) and the YOULA parameter poles (spec(A_Q)),
- the closed-loop state-estimator poles and the YOULA parameter poles are uncontrollable by e,
- the closed-loop state-feedback poles and the YOULA parameter poles are unobservable from ε_y.

$\Rightarrow$ The transfer function from e to ε_y always vanishes.

Observer-based ... - Principle

The basic idea is to express the controller state equation ($\dot{x_K} = A_K x_K + B_K y$) as an LUENBERGER observer of the variable $z = Tx$:

$$x_K = \widehat{z} \tag{3}$$

According to LUENBERGER's formulation, this problem can be stated as the search of

$$T \in \mathbb{R}^{n_K \times n},\ F \in \mathbb{R}^{n_K \times n_K},\ G \in \mathbb{R}^{n_K \times p}$$

such that:

$$\dot{\widehat{z}} = F\widehat{z} + G(y - Du) + TBu \tag{4}$$

is an (asymptotic) observer of the variable z, i.e. such that:

$$TA - FT = GC, \quad \text{and } F \text{ stable }. \tag{5}$$

The solution consists to solve in T the generalized RICCATI equation:

$$\boxed{A_K T - T(A + BD_K C) - TBC_K T + B_K C = 0\ .}\ . \tag{6}$$

Observer-based ... - Principle

or: $[-T \quad I]A_{cl}\begin{bmatrix} I \\ T \end{bmatrix} = 0$ with: $A_{cl} := \begin{bmatrix} A + BD_KC & BC_K \\ B_KC & A_K \end{bmatrix}$. (7)

Then: $\boxed{G = B_K - TBD_K}$ and $\boxed{F = A_K - TBC_K}$.

T is the $n_K \times n$ matrix of the n_K linear combinations of the n plant states which are observed by the n_K controller states. Then:

- if $n_K > n$, one can find an exact observer-based realization involving a YOULA parameter of order $n_K - n$,
- if $n_K = n$, the YOULA parameter is static ($=$ the controller direct feedthrough),
- if $n_K + p \geq n$, (p is the length of y), one can estimate plant states by $\widehat{x} = H_1T + H_2C$ provided : $H_1T + H_2C = I_n$,
- if $n_K + p < n$,, one can estimate the state of a plant reduced model (see application).

Observer-based ... - Principle

The RICCATI equation is solved by invariant subspaces approach:

- finding a n-dimensional invariant subspace $\mathcal{S} := Range(U)$ of the closed-loop system matrix A_{cl}, that is,

$$A_{cl}U = U\Lambda$$

  This subspace is associated with a set of n eigenvalues, spec(Λ), among the $n + n_K$ eigenvalues of A_{cl}.

- partitioning the vectors U which span this subspace conformably to the partitioning in (7).

$$U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}, \quad U_1 \in \mathbb{R}^{n\times n}.$$

- computing the solution: $\boxed{T = U_2 U_1^{-1}.}$

Observer-based ... - Principle

Problem solving RICCATI equation: among $n + n_K$ closed-loop eigenvalues, choose:

- n (auto-conjugate) eigenvalues for spec$(A - BK_c)$,
- n (auto-conjugate) eigenvalues for spec$(A - K_fC)$,
- $n_K - n$ (auto-conjugate) eigenvalues for spec(A_Q).

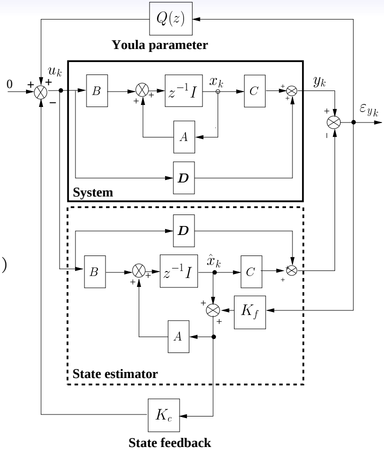
Good sense rules:

- affect the fastest poles to spec(A_Q) in such a way that the Youla parameter acts as a direct feedthrough in the compensator,
- assign to spec$(A - BK_c)$ the n closed-loop poles which are the "nearest" from the n plant poles in order to respect the dynamic behavior of the physical plant and reduce the state-feedback gains,
- assign fast closed-loop poles to spec$(A - K_fC)$ to have an efficient state estimator.

Observer-based ... - Discrete-time case

The observer based realization of the compensator reads (estimator form):

$$\left\{\begin{array}{rcl} \hat{x}_{k+1} & = & A\hat{x}_k + Bu_k + AK_f(y_k - C\hat{x}_k - Du_k) \\ x_{Q\,k+1} & = & A_Q x_{Q\,k} + B_Q(y_k - C\hat{x}_k - Du_k) \\ u_k & = & -K_c\hat{x}_k + C_Q x_{Q\,k} + \ldots \\ & & \ldots(D_Q - K_cK_f)(y_k - C\hat{x}_k - Du_k) \end{array}\right.$$



The discrete YOULA parameterization on the estimator LQG structure.

Outline

Observer-based ... - Illustration 1: state monitoring

Consider the plant $G_0(s)$ and the compensator $K_1(s)$ (positive feedback):

$$G_0(s) = \frac{1}{s^2-1} \quad \text{(launcher)} \quad \equiv \quad \left[\begin{array}{c} \dot{\psi} \\ \ddot{\psi} \\ \hline \psi \end{array}\right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right] \left[\begin{array}{c} \psi \\ \dot{\psi} \\ \hline \beta \end{array}\right],$$

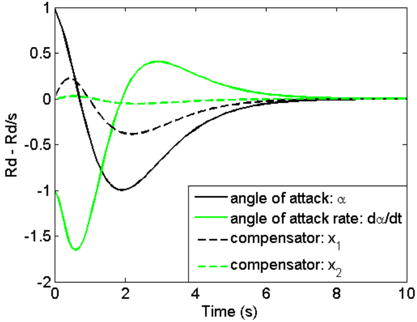
$$K_1(s) = -\frac{s^2+27s+26}{s^2+7s+18} \quad \equiv \quad \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \hline \beta \end{array}\right] = \left[\begin{array}{cc|c} 0 & -18 & 1 \\ 1 & -7 & 0 \\ \hline -20 & 132 & -1 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \\ \hline \psi \end{array}\right].$$

Choice: spec$(A - BK_c) = \{-1.,\ -2.\}$, spec$(A - K_f C) = \{-2.,\ -2.\}$

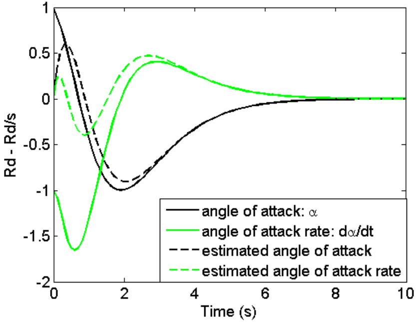
$$\Longrightarrow \quad K_c = [3 \quad 3];\ K_f = [4 \quad 5]^T;\ Q = -1\,.$$

$$K_1(s) \quad \equiv \quad \left[\begin{array}{c} \dot{\hat{\psi}} \\ \dot{\hat{\dot{\psi}}} \\ \hline \beta \end{array}\right] = \left[\begin{array}{cc|c} -4 & 1 & 4 \\ -6 & -3 & 4 \\ \hline -2 & -3 & -1 \end{array}\right] \left[\begin{array}{c} \hat{\psi} \\ \hat{\dot{\psi}} \\ \hline \psi \end{array}\right] \text{ (observer-based form).}$$

Observer-based ... - Illustration 1: state monitoring



Responses to initial conditions on launcher states - companion realization of $K_1(s)$.



Responses to initial conditions on launcher states - observer-based realization of $K_1(s)$.

Outline

Observer-based ... - Illustration 2: controllers switch

Let us consider a second stabilizing and wide-band controller:

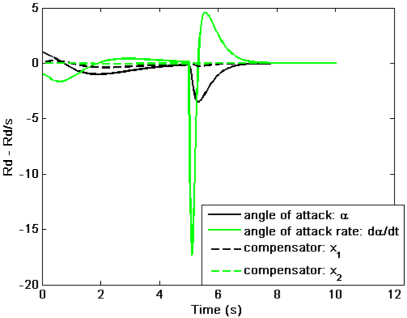
$$K_2(s) = -\frac{1667s + 2753}{s^2 + 27s + 353} \quad \equiv \quad \left[\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \hline \beta \end{array}\right] = \left[\begin{array}{cc|c} 0 & -353 & 1 \\ 1 & -27 & 0 \\ \hline -1667 & 42260 & 0 \end{array}\right] \left[\begin{array}{c} x_1 \\ x_2 \\ \hline \psi \end{array}\right] .$$

and let us assume that the control law must switch from controller K_1 to controller K_2 at time $t = 5\,s$.
The new closed loop dynamics reads:
spec$(A_{cl}) = \{-3, \quad -4, \quad -10 + 10\,i, \quad -10 - 10\,i\}$.
Then assigning -3 and -4 to the state-feedback dynamics, we get:

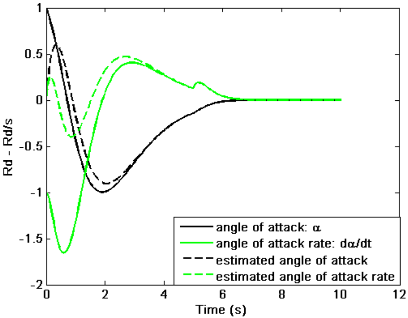
$$K_c = [13 \quad 7];\; K_f = [20 \quad 201]^T;\; Q = 0\,.$$

At $t = 5\,s$, state vector of $K_2(s)$ is initialized with the current state of $K_1(s)$.

Observer-based ... - Illustration 2: controllers switch

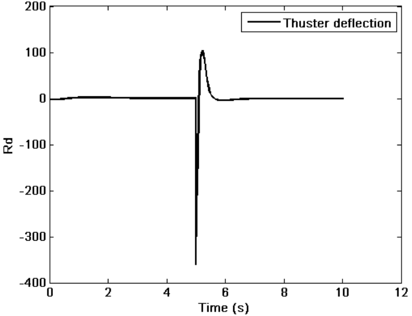


Responses to initial conditions and switch from $K_1(s)$ to $K_2(s)$ at time $t = 5\,s$ - companion realizations of $K_i(s)$.

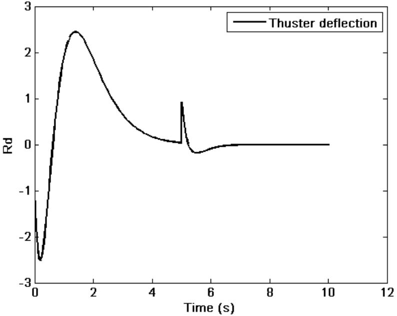


Responses to initial conditions and switch from $K_1(s)$ to $K_2(s)$ at time $t = 5\,s$ - observer-based realizations of $K_i(s)$.

Observer-based ... - Illustration 2: controllers switch



Responses of the control signal δ to initial conditions and switch from $K_1(s)$ to $K_2(s)$ at time $t = 5\,s$ - companion realizations of $K_i(s)$.



Responses of the control signal δ to initial conditions and switch from $K_1(s)$ to $K_2(s)$ at time $t = 5\,s$ - observer-based realizations of $K_i(s)$.

Outline

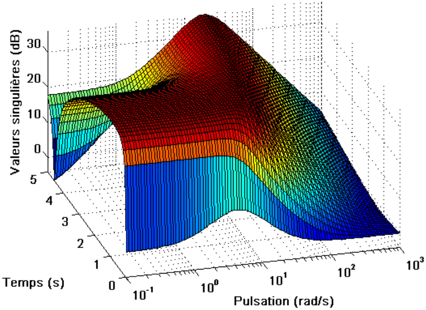
Observer-based ... - Illustration 3: smooth gain scheduling

Now, the objective is to interpolate the controller from K_1 to K_2 over $5\,s$ (starting from $t = 0$).

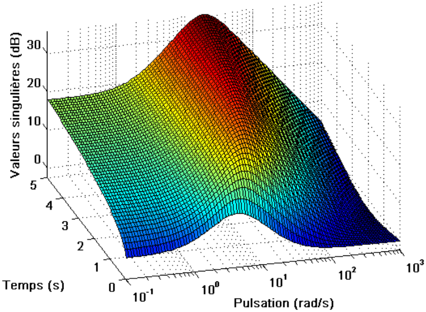
The linear interpolation of the 4 state-space matrices of companion realizations of K_1 and K_2 provides a non-stationary controller $K(s,t)$ whose frequency response w.r.t. time t is non-smooth. Furthermore, $K(s,t)$ does not stabilize the nominal plant $G_0(s)$ for $0.1 < t < 4.6\,(s)$.

In comparison, the interpolation of the 4 state-space matrices of observer-based realizations of K_1 and K_2 is quite smooth. $\Rightarrow$ $K_{observer-based}(s,t)$ stabilizes $G_0(s)\ \forall\, t$.

Observer-based ... - Illustration 3: smooth gain scheduling



$K(s,t)$: singular value w.r.t time.

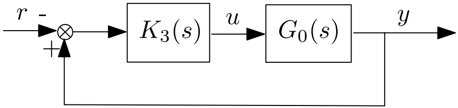


$K_{observer-based}(s,t)$: singular value w.r.t time.

Outline

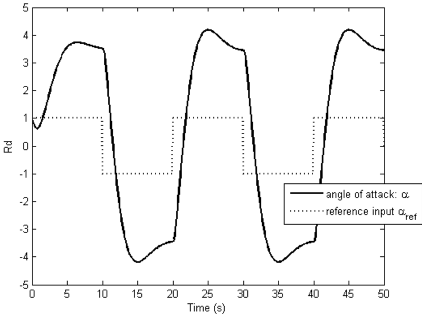
Observer-based ... - Illustration 4 : reference input plugging

Let us consider a third controller designed for disturbance rejection:
$K_3(s) = -\frac{50s+70}{s^2+10s+50}$.
One can take into account the reference input $r(t)$ according to the well-know feedback servo-loop scheme:

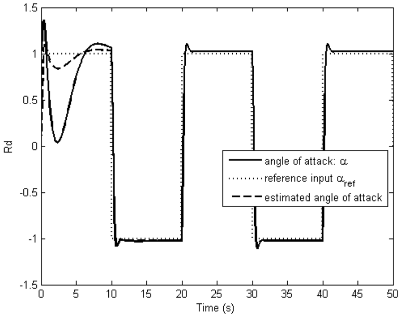


Here $r(t) = \alpha_{ref}(t)$. Considering the observer based realization, it seems quite logical to send in the state feedback gain K_c the difference between the estimated $\widehat{x} = [\widehat{\alpha} \quad \widehat{\dot{\alpha}}]^T$ and the reference state $x_{ref} = [\alpha_{ref} \quad 0]^T$. Then the state-estimation dynamics is uncontrollable by α_{ref} and that can be used to improve response to reference input (all the more if the state-estimation dynamics is slow).

Observer-based ... - Illustration 4: reference input plugging



Time domain responses of classical-loop.



Time domain responses using observed-based realization (fast dynamics ($6.36\,rd/s$) is assigned to spec$(A - BK_c)$).

Outline

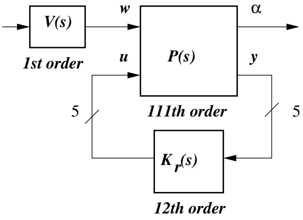
Observer-based ... - Low order compensator case

The separation principle is sensitive to the difference between the real plant and the onboard model.
$\Rightarrow$ for a given reduced n_K order compensator, one can select a n_K-order onboard model:

- this onboard model must be as representative as possible of the real plant (modelled by a full order model of order $n > n_K$),
- this onboard can also include a model of disturbances, mainly if these disturbances are more influent on the closed-loop behavior than secondary dynamics of the real plant.

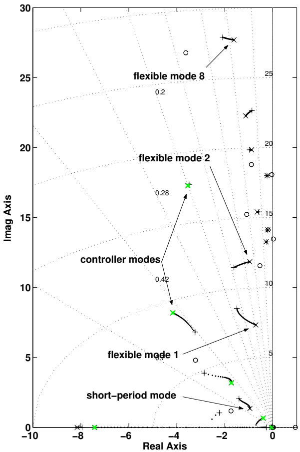
The on-board model selection is a little bit "touchy" and is based on a practical know-how.
The application presented here concern wind monitoring from a longitudinal flight control of a flexible aircraft.

Application: wind monitoring using flight control law



Validation model $P(s)$:

- 1 short-period mode,
- 28 flexible modes,
- 40 secondary poles (actuators, ...).
- a 13-th order turbulence



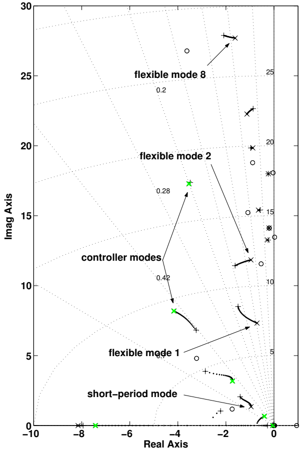
$-K_r(s)P_{22}(s)$ root locus.

12-th order onboard model P_o computation

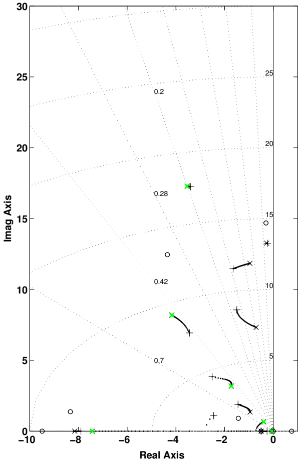
From a a bloc-diagonal realization of P:

- the short-period mode and the first 4 flexible modes are kept,
- the 40-th order subspace associated with the fast and damped poles is balanced and truncated to order 2,
- the 13-th order turbulence model is neglected but a 2-nd order DRYDEN filter is introduced on the wind input to represent roughly this turbulence model and the filter $V(s)$,
- using the output matrix, a change of variable is finally performed in such a way: the first 7 states of the onboard model P_o correspond to the 6 outputs (angle-of-attack, 5 measurements) and the wind w (that is: the output of the DRYDEN filter).

Onboard model P_o validation



$-K_r(s)P_{22}(s)$ root locus.

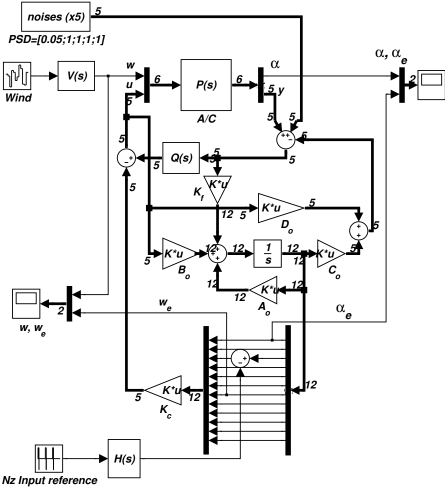


$-K_r(s)P_{o22}(s)$ root locus.

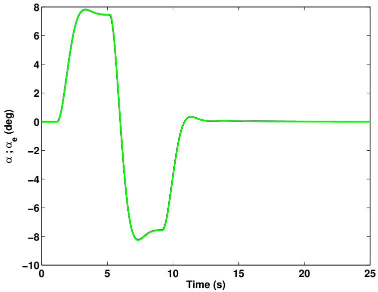
Choice of the observer-based realization

Choice in the combinatory set of solutions: among the 24 closed-loop eigenvalues, the 12 eigenvalues associated with spec$(A_o - B_oK_c)$ are in fact the 12 eigenvalues which are located on the branches starting from the 12 open-loop eigenvalues of the onboard model $P_o(s)$ (black $\times$), in the root locus of $-K_r(s)P_{o22}(s)$.
$\Rightarrow$ such a choice can be easily systematized by an simple procedure.

Simulation

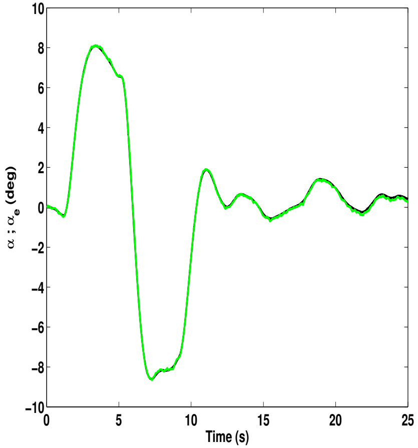


Simulator with full order model.

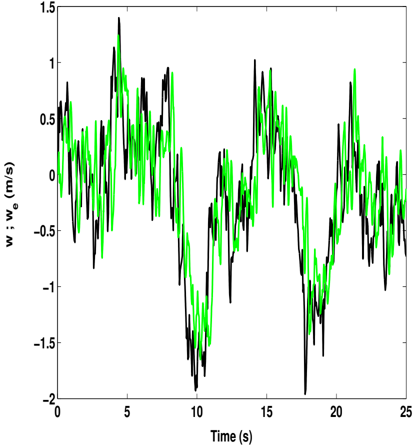


Black: $\alpha(t)$; green: $\alpha_e(t)$ (without disturbances: $W = V_q = V_{N_z} = 0$).

Simulation results

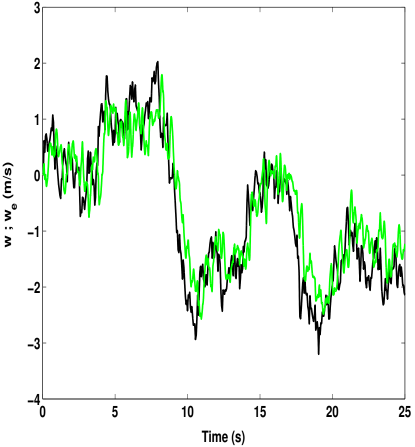


Black: $\alpha(t)$; green: $\alpha_e(t)$.

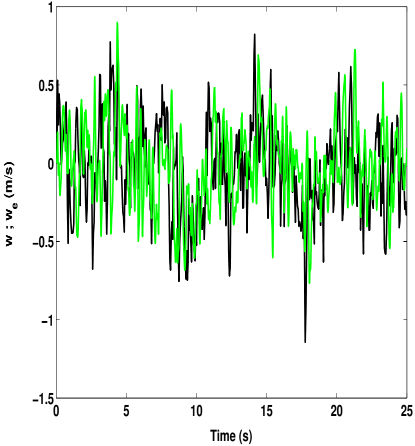


Black: $w(t)$; green: $w_e(t)$.

Robustness to wind model



Black: $w(t)$; green: $w_e(t)$ (with $V(s) = 1/(s+0.2)$).



Black: $w(t)$; green: $w_e(t)$ (with $V(s) = 1/(s+5)$).

Outline

Observer-based... - Concusions

- The observer-based realization of a given controller can be used to estimate or to monitor some unmeasured plant states or external disturbances.
- Such a monitoring can be used to perform in-line or off-line analysis (supervising controller modes, capitalizing flight data to improve disturbance modeling, ...) saving computational effort,
- Application to (re)-tuning of observer-based controllers (F. Delmond et Al.: AIAA JGCD 2004, Vol 27, No. 4).
- Application to disturbance monitoring and fault detection for a launcher (F. O. Ramos and D. Alazard: AIAA GNC2009).

Perspectives:

- a (more) general and optimal (w.r.t. estimates quality) procedure to select the onboard model in the combinatory set of solutions ?
- sensitivity of wind and angle of attack estimates to parametric uncertainties ?

Outline

Outline

Cross Standard Form (CSF).

OBJECTIVES: to solve a very practical problem:

- Is it possible to improve a given controller $K_0(s)$ to meet additional H_2 or H_∞ specifications ?

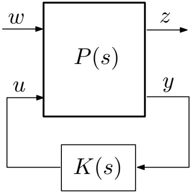
OR (equivalently):

- Is it possible to take into account $K_0(s)$ in a standard H_2 or H_∞ control problem ?

$\Rightarrow$ a very close link with INVERSE OPTIMAL CONTROL PROBLEM.
Aeronautical application example: a low-order controller $K_0(s)$, designed upon a particular industrial know-how or good sense rules, is given. The load-in-turbulence index performance is the H_2 norm between the turbulence input and the load output. A solution to the H_2 inverse optimal control problem would allow to take into account K_0 in a standard H_2 problem in order to be mixed with the H_2 load alleviation problem.

Reminder on H_∞ standard problem

Standard problem: an augmented model embedding the model $G(s)$ (transfer from u to y) and frequency-domain specifications (exogenous input w and controlled variables z).



Standard interconnection:
$F_l(P, K)$

Representations:

- $P(s) = \begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{uw}(s) & P_{yu}(s) \end{bmatrix}$

- $\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}$

.

H_∞ control problem: find a stabilizing controller $K(s)$ such that:

$$\boxed{\|F_l(P(s), K(s))\|_\infty \leq \gamma \quad \text{for a given scalar } \gamma}$$

with: $\|F(s)\|_\infty = \max_\omega \bar{\sigma}(F(j\omega))$

where $\bar{\sigma}(M)$ is the highest singular value of matrix M.

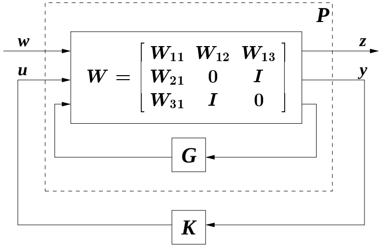
CSF and inverse optimal control problem.

"We know the controller, we seek the performance index..."

- inverse LQ pb: KALMAN (ASME 64'), MOLINARI (TAC 73'): state feedback.
- inverse H_∞ pb: FUJY, KHARGONEKAR (CDC 88'): state feedback; KHARGONEKAR, DOYLE (MCSS 88'): mixed sensibility scheme, siso case.

General case: output feedback
Standard control problem set-up around the plant $G(s) = P_{yu}(s) \quad \Rightarrow$

- N. SEBE (CDC 2001): find $G(s)$, given $W(s)$ and $K(s)$.



General interconnection scheme.

Cross Standard Form (CSF) - definitions

Given $G(s)_{p\times m}$ (order n) and $K_0(s)_{m\times p}$ (order n_K):

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}, \quad \begin{bmatrix} \dot{x_K} \\ u \end{bmatrix} = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \begin{bmatrix} x_K \\ y \end{bmatrix}.$$

Inverse H_2 problem: find a standard control problem $P(s)$ such that:

- $P_{yu}(s) = G(s)$,
- K_0 stabilizes $P(s)$ and $K_0(s) = \arg\min_{K(s)} \|F_l(P(s), K(s))\|_2$,

Inverse H_∞ problem: find a standard control problem $P(s)$ such that:

- $P_{yu}(s) = G(s)$,
- K_0 stabilizes $P(s)$ and $K_0(s) = \arg\min_{K(s)} \|F_l(P(s), K(s))\|_\infty$.

Cross Standard Form: a standard control problem $P(s)$ such that:

- C1: $P_{yu}(s) = G(s)$,
- C2: K_0 stabilizes $P(s)$ and C3: $F_l(P(s), K_0(s)) = 0$,
- C4: K_0 is the unique solution of the optimal H_2 or H_∞ problem $P(s)$,

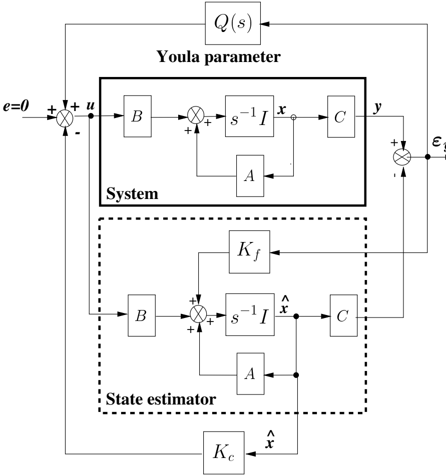
Cross Standard Form (CSF): case $n_K \geq n$

CSF BASED ON THE OBSERVER-BASED REALIZATION of $K_0(s)$.

Step 1 From $G(s)$ and $K_0(s)$, compute:

- the state feedback gain K_c,
- the state estimator gain K_f,
- the dynamic YOULA parameter $Q(s)$ (order $n_K - n$ defined by a minimal state space realization: A_Q, B_Q, C_Q, D_Q)

of the observer based realization of K_0.

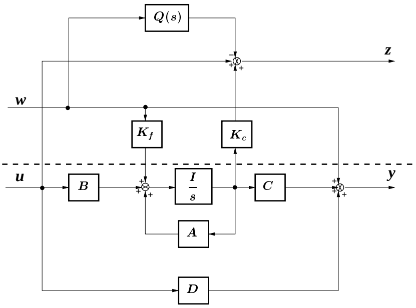


Cross Standard Form (CSF): case $n_K \geq n$

Step 2 From $G(s)$, K_c, K_f and $Q(s)$, build the CSF:

$$P_{CSF} := \left[\begin{array}{cc|c|c} A & 0 & K_f & B \\ 0 & A_Q & B_Q & 0 \\ \hline K_c & -C_Q & -D_Q & I_m \\ \hline C & 0 & I_p & D \end{array}\right]$$

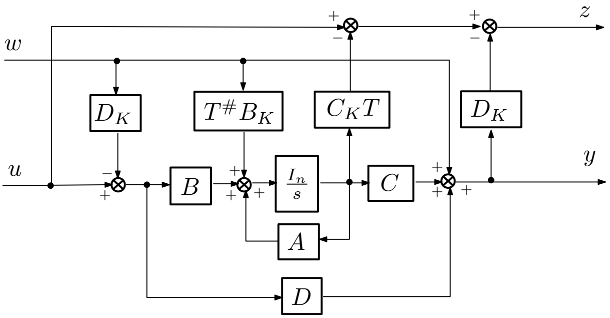
(see D. Alazard et al. in SSSC'04; Oaxaca, Mexico).



Block diagram of $P_{CSF}(s)$.

CSF: generalization (case $n_K \leq n$)

Main result: given $G(s)$ and $K_0(s)$, a CSF is:



where $T_{n_K \times n}$ is the solution a the RICCATI equation (GNARE):

$$[-T \quad I] \begin{bmatrix} A + B(I - D_K D)^{-1} D_K C & B(I - D_K D)^{-1} C_K \\ B_K (I - D D_K)^{-1} C & A_K + B_K D (I - D_K D)^{-1} C_K \end{bmatrix} \begin{bmatrix} I \\ T \end{bmatrix} = 0$$

and $T^{\#}$ is a right-inverse of T.

CSF: generalization (case $n_K \leq n$)

that is: $$\boxed{P_{CSF}(s) := \left[\begin{array}{c|cc} A & T^{\#}B_K - BD_K & B \\ \hline -C_KT - D_KC & D_KDD_K - D_K & I_m - D_KD \\ C & I_p - DD_K & D \end{array}\right]}$$

Proof: with the state mapping $\mathcal{M} = \mathcal{M}^{-1} = \begin{bmatrix} I_n & 0 \\ T & -I_{n_K} \end{bmatrix}$,

$F_l(P_{CSF}, K_0)$ reads [1]:

$$\left[\begin{array}{cc|c} A + BJ_m(D_KC + C_KT) & -BJ_mC_K & T^{\#}B_K \\ 0 & A_K + (B_KD - TB)J_mC_K & 0 \\ \hline 0 & -C_K & 0 \end{array}\right]$$

That is $\boxed{F_l(P_{CSF}(s), K_0(s)) = 0}$.

[1] with $J_m = (I_m - D_KD)^{-1}$ and $J_p = (I_p - DD_K)^{-1}$.

CSF: generalization (case $n_K \leq n$)

for a given (G, K_0):

- a combinatory set of CSF exits according to the choice of n eigenvalues among $n + n_K$ closed-loop eigenvalues in solving the RICCATI equation by invariant subspaces approach and according to the choice of the right inverse $T^{\#}$ of T,
- but for a CSF, the optimal H_2 or H_∞ controller is unique (and equal to $K_0(s)$), (see F. DELMOND ET AL. in IJC'06),
- existence of a CSF:
  if $\exists$ a CSF then $\exists$ T solution of the RICCATI equation (GNARE),
- the generalization to the case $n_K \leq n$ encompasses the previous CSF formulation based on the observer-based realization of $K_0(s)$ and includes also 2 d.o.f. controllers.

CSF numerical example: (case $n_K \leq n$)

Let us consider the system $G_0(s) =$ and initial stabilizing controller $K_0(s)$

$$G_0(s) = \frac{1}{s^2-1} := \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}\right], \quad K_0(s) = \frac{-23s-32}{s+12} := \left[\begin{array}{c|c} -12 & 4 \\ \hline 61 & -23 \end{array}\right].$$

Closed-loop dynamics: $\{-1 \pm i, \ -10\}$.

The only real solution T of $\textsc{Riccati}$ equation is :
$T = [0.32787 \quad -0.032787]$.
Let us choose $T^{\#} = T^{+}$, then the corresponding CSF reads:

$$P_{CSF} := \left[\begin{array}{cc|cc} 0 & 1 & 12.079 & 0 \\ 1 & 0 & 21.792 & 1 \\ \hline 3 & 26 & 23 & 1 \\ 1 & 0 & 1 & 0 \end{array}\right].$$

$\Rightarrow$ the optimal H_∞ controller is: $K_\infty(s) = -23\frac{(s+1.391)(s+2.079)}{(s+12)(s+2.079)}$.
Obviously, K_∞ is not minimal and $K_\infty = K_0$.

CSF: generalization (case $n_K \leq n$)

MATLAB H_∞ solvers provide a n-th order controller with $n - n_K$ pole/zero cancellations. It is possible to prescribe the values of cancellations and it is recommended to assign these values far in left half s-plane.

Indeed, $T^{\#}$ can be parameterized : $T^{\#} = T^{+} + T^{\perp}X$ where $T^{\perp} = \ker(T)$ and X is a $(n - n_K) \times n_K$ matrix of free parameters.

X allows the $n - n_K$ pole/zero cancellation to be assigned in the s-plane. It can be shown that X^T is the state feedback, on the pair $({T^{\perp}}^T(A + BJ_mD_KC)^TT^{\perp},\ (B_KJ_pCT^{\perp})^T)$, assigning the dynamics to the prescribed values.

CSF numerical example: (case $n_K \leq n$)

```
a=[0 1;1 0];b=[0;1];c=[1 0];d=0;
AK=-12;BK=4;CK=61;DK=-23;
T=cor2tfga(ss(a,b,c,d),ss(AK,BK,CK,DK));
Nt=null(T);
AA=Nt'*(a+b*DK*c)'*Nt;
BB=Nt'*c'*BK';
X=place(AA,BB,[-100]);X=X';
Tm1=pinv(T)+Nt*X;
plant=ss(a,[Tm1*BK-b*DK b],[-CK*T-DK*c;c],...
... [-DK+DK*d*DK,eye(size(d,2))-DK*d;eye(size(d,1))-d*DK d]);
K3=hinfsyn(plant,1,1);
zpk(K3)
```

Zero/pole/gain:

$$\frac{-23\ (s+1.391)\ (s+100)}{(s+100)\ (s+12)}$$

Outline

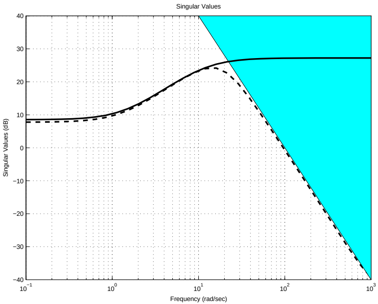
Illustration 1: Improving K_0 with frequency domain specification

In fact, K_0 has been designed to assign the dominant closed-loop eigenvalues to $-1 \pm i$:

CL dynamics: $= \{-1+i,\ -1-i,\ -10\}$.

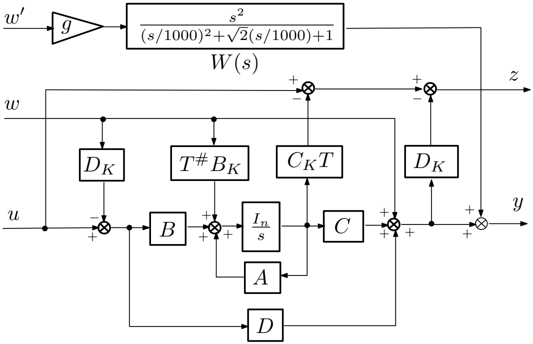
The controller must now fit the template $\Rightarrow$.

Solution: a CSF augmented with an H_∞ weight.



Frequency-domain responses (magnitude) of $K_0(s)$ (solid line) and $K(s)$ (dashed line) and template (grey patch).

Illustration 1: Improving K_0 with frequency domain specification

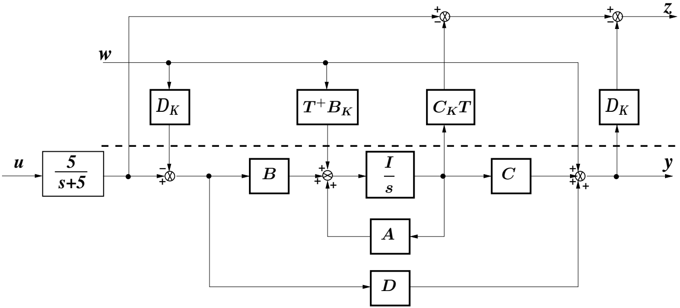


Augmented Cross Standard Form to take into account roll-off specification (with $T^{\#} = [27.5 \quad 244.5]^T$). g is tuned by a try-an error procedure.

Outline

Illustration 2: assigning closed-loop dominant dynamics

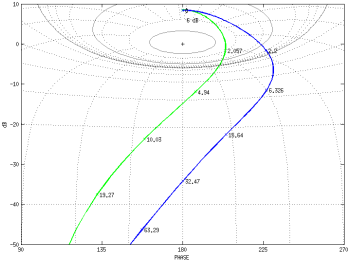
Taking into account actuator dynamics : $A(s) = \frac{5}{s+5}$.



Cross Standard Form augmented with actuator dynamics.

$$\Rightarrow K(s)$$

Illustration 2: assigning closed-loop dominant dynamics



NICHOLS: $-K_0(s)A(s)G(s)$ (green) $-K(s)A(s)G(s)$ (blue).

Closed-loop dynamics:

- with K_0: $\{-0.77 \pm 1.59\,i,\ -2.45,\ -13\}$,
- with $K(s)$: $\{-1 \pm i,\ -5,\ -10,\ -100\}$.

Outline

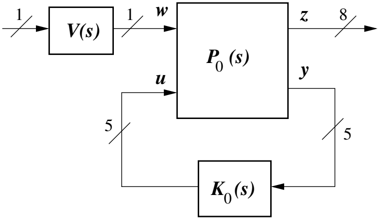
Improving flight control law for load alleviation.

$P_0(s)$: longitudinal model of the A/C with:

- 29 states: 1 rigid (short-term) mode + 8 flexible modes + 11 fast and damped poles (actuator dynamics and aerodynamic lags),
- 5 control signals u (inner and outer elevators and inner, middle and outer ailerons)
- 5 measurements y (pitch rate q and 4 measurements of vertical acceleration n_z in various locations of the A/C),
- 1 turbulence input w fitted with a DRYDEN filter $V(s)$,
- 8 load outputs z (4 bending moments and 4 shear forces at critical points in the structure).

$K_0(s)$: 11th-order initial controller designed (upon a practical industrial know-how) to assign the short-term mode and to damp the first flexible mode in spite of parametric uncertainties.

Improving flight control law for load alleviation.



Initial data.

OL and CL load performances:

$$\|P_0(1:8,1)V\|_2 = 2.05\,10^7 \ ,$$

$$\|F_l(P_0, K_0)V\|_2 = 1.91\,10^7 \ .$$

State space realizations of P_0 and K_0:

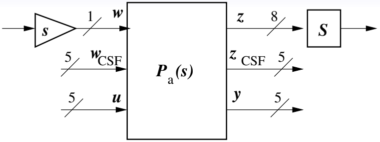
$$P_0(s) := \left[\begin{array}{c|cc} A & B_1 & B \\ \hline C_1 & 0 & D_{12} \\ C & D_{21} & D \end{array}\right] ,$$

$$K_0(s) := \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] .$$

From T, the following augmented control problem P_a is set up:

$$\left[\begin{array}{c|ccc} A & B_1 & T^+B_K - BD_K & B \\ \hline C_1 & 0 & 0 & D_{12} \\ -C_KT - D_KC & 0 & -D_K + D_KDD_K & I_m - D_KD \\ C & D_{21} & I_p - DD_K & D \end{array}\right]$$

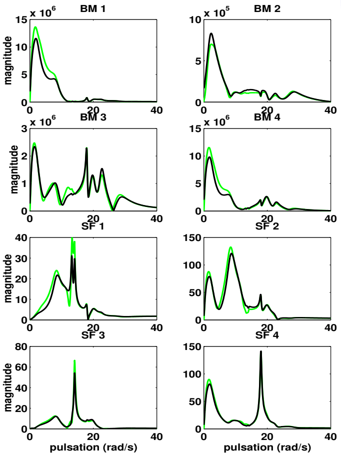
Improving flight control ....



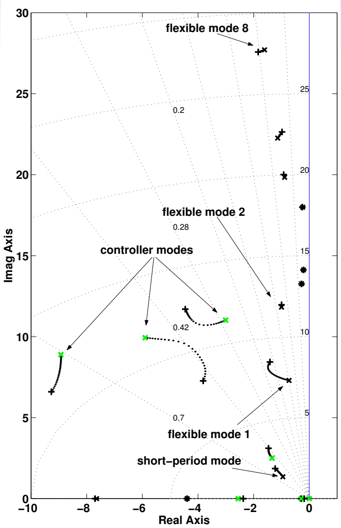
Final problem for load alleviation.

s and S: static scalings tuned to meet a trade-off between load alleviation and initial dynamic behavior.
Final H_2 design and balanced reduction $\Rightarrow$ 12th-order controller $K_r(s)$:

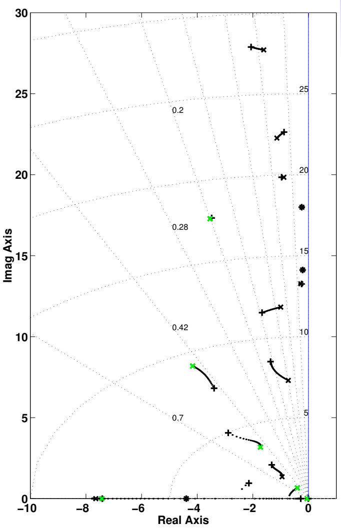
$$\|F_l(P_0, K_r)V\|_2 = 1.57\,10^7 \ (-18\%).$$



Load frequency-domain responses: K_0 (green), K_r (black).



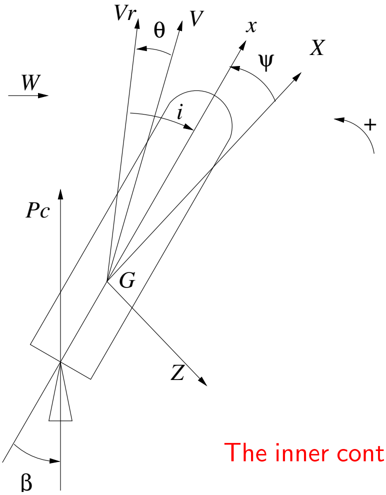
Root locus of $P_0(s)$ and $K_0(s)$.



Root locus of $P_0(s)$ and $K_r(s)$.

Outline

Launcher control loop design.



Notations:

- i: launcher angle of attack,
- ψ: yaw attitude,
- β: thruster angle of deflection,
- V: launch velocity,
- W: wind velocity,
- $\dot{z}$: launcher lateral drift rate.

Angle of attack equation:
$i = \psi + \frac{\dot{z}-w}{V}$

The inner control loop is a pure rejection problem, not a tracking problem.

Launcher control loop design.

- 2 measurements: ψ,$\dot{\psi}$,
- 1 control signal: β,
- 1 disturbance: w,
- at least 1 controlled variable: i (reduction of loads on the structure).

Synthesis model = 3^{rd} order rigid model: $x_r = [\psi,\ \dot{\psi},\ \dot{z}]$

$$\left[\begin{array}{c} \dot{x^r} \\ \hline i \\ \dot{z} \\ \hline \psi \\ \dot{\psi} \end{array}\right] = \left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right] \left[\begin{array}{c} x^r \\ \hline w \\ \hline \beta \end{array}\right] \quad \text{(standard problem)}$$

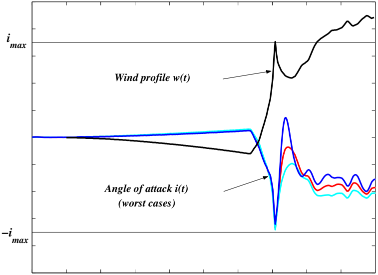
Validation model = 3^{rd} order rigid model + sensor and actuator dynamics + 5 bending modes.

Launcher control loop design.

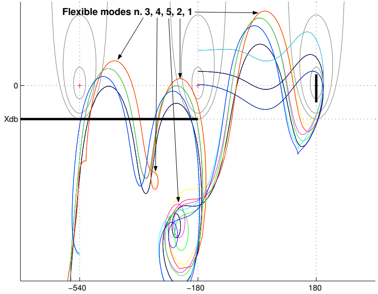
A multi-objective problem:

- time domain performance: angle of attack response to a typical wind profile: $w(t)$ is constrained $-i_{max} < i(t) < i_{max} \; \forall \; t$,
- frequency domain specifications: stability margins measured on NICHOLS plots:
  - low frequency gain margin,
  - high frequency gain margin,
  - 1 sampling period delay margin,
  - maximal gain level for bending mode number,2 ,3 ,...., (phase control of the first bending mode),
- parametric robustness: all the templates must be satisfied for a set of identified worst-cases selected in the uncertain parameter space:
  - aerodynamic coefficient (yaw axis),
  - thruster efficiency,
  - flexible mode characteristics: pulsations, modal shapes,...,
- consumption reduction.

Launcher control loop design - Templates to be satisfied.



Angle of attack response (worst cases).



NICHOLS plot for worst cases.

Launcher control loop design - stationary control design.

A multi-step control design based on the Cross Standard form:

- performance: LQG/LTR design on the rigid model to meet performance and stability margins around the rigid dynamics (that is: low and high frequency gain and delay margins),
- frequency domain robustness: H_∞ design to meet robustness specification on bending modes:
  - in fact, the phase of the first flexible mode is naturally controlled (positivity) due to a judicious location of the sensors along the fuselage.
  - the attenuation of higher flexible modes (2,3, ...) is taken into account with a roll-off specification on the control signal. The cut-off frequency must be tuned between the first flexible mode and the second flexible mode and must be selective enough to attenuate higher flexible modes without too much phase lag at the first flexible mode frequency.

Launcher control loop design - Pure performance synthesis.

1. LQ synthesis on the 3^{rd} order rigid model:

$$J = \int_0^\infty \left(\alpha \dot{z}^2 + i^2 + r u^2\right) dt.$$

$$\Rightarrow \quad K = [K_\psi \; K_{\dot\psi} \; K_{\dot z}] \quad \Rightarrow \quad \beta = K_\psi(\psi_{\text{ref}} - \psi) - K_{\dot\psi}\dot\psi - K_z \dot z$$

2. Rough wind modelling:

$$\dot W = -\lambda_W W + B_W \quad \Rightarrow \quad X_{aug} = [\ \psi \quad \dot\psi \quad \dot z \quad W\ ]^T$$

$$\left[\begin{array}{c} \dot X_{aug} \\ \hline i \\ \dot z \\ \hline \psi \\ \dot\psi \end{array}\right] = \left[\begin{array}{c|c|c} A_{aug} & B_{1_{aug}} & B_{2_{aug}} \\ \hline C_{1_{aug}} & D_{11_{aug}} & D_{12_{aug}} \\ \hline C_{2_{aug}} & D_{21_{aug}} & D_{22_{aug}} \end{array}\right] \left[\begin{array}{c} X_{aug} \\ \hline B_W \\ \hline \beta \end{array}\right]$$

Launcher control loop design - Pure performance synthesis.

3. Augmented state-feedback gain:

$$\psi_{ref} = \frac{W-\dot{z}}{V} \text{ to cancel } i = \psi + \frac{\dot{z}-W}{V}$$

$$\beta = K_\psi(\frac{W-\dot{z}}{V} - \psi) - K_{\dot{\psi}}\dot{\psi} - K_z\dot{z} \quad \Rightarrow \quad K_{aug} = \left[\begin{array}{cc} K & -\frac{K_\psi}{V} \end{array} \right]$$

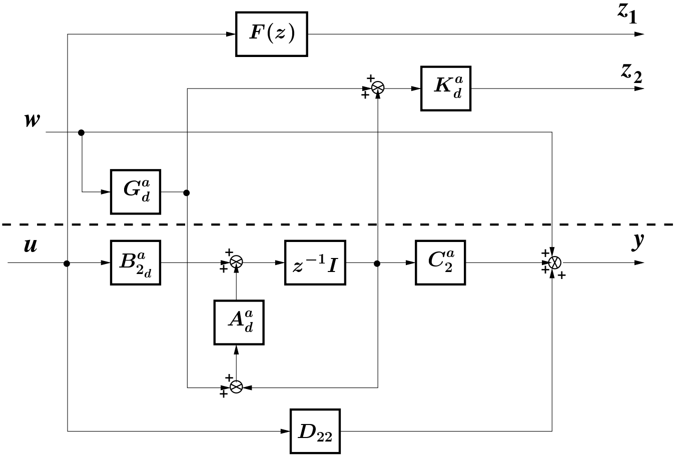
(the term $\dot{z}/V$ is ignored to ensure closed-loop stability)

5. KALMAN: gain computation

$$W_f = B_{1_{aug}} B_{1_{aug}}^T + q B_{2_{aug}} B_{2_{aug}}^T, \quad V_f = v \left[\begin{array}{cc} 1 & 0 \\ 0 & \omega^2 \end{array} \right]$$

4. remark: In fact, model and weighting matrices are discretized assuming a zero-holder on the input (see. O. Voinot *et al.*, Control Engineering Practice 2003).

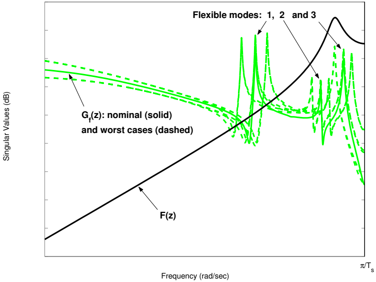
Launcher control loop design - final stationary design.



Final standard discrete-time H_∞ problem: $P(z)$.

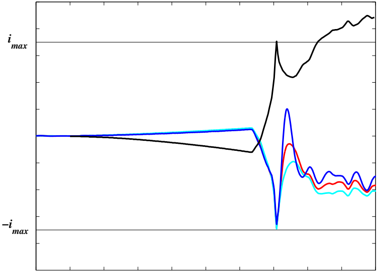
Launcher control loop design - final stationary design.

Roll-off filter specification ($F(z)$): $\Rightarrow$ trade-off between high flexible modes attenuation and low stability margins.

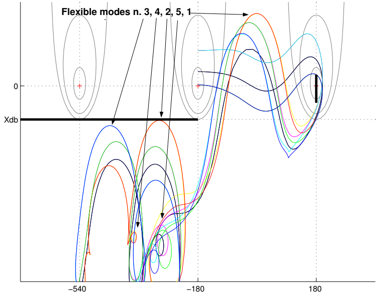


$F(z)$ frequency response: peak variations of flexible modes 2 and 3 are framed by the filter response.

Launcher control loop design - final stationary result.



Angle of attack.



NICHOLS plots (worst cases).

Outline

Conclusions.

Cross Standard Form (i.e. : a standard plant whose unique H_2 or H_∞ optimal controller is equal to a given controller) is now generalized to arbitrary order controller case :

- $\Rightarrow$ a general solution to the inverse optimal control problem (Reverse engineering),
- $\Rightarrow$ allow to mix (indirectly) various approaches and to solve practical multi-channel control problems. The case $(n_K < n)$ is very attractive form a practical point of view: allows to take into account an initial controller, based on practical (industrial) know-how, in a multi-objective design procedure.

Open problems:

- The structure of initial controller K_0 is lost.
- Final design sensitivity w.r.t. the choice of the CSF ?
- Choice of scalings between the two channels of the final standard control problem?