Exercises on H_∞ control with Matlab

1 H_∞ design

Let us consider the second order model of an unstable aerospace vehicle:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 - 1}$$

The objective is to design an H_∞ feedback controller $K(s)$ to meet the template depicted in Figure 1 on the sensitivity function S. The model G is also subject to an additive uncertainty Δ whose an upper bound in the frequency-domain is plotted in Figure 2.

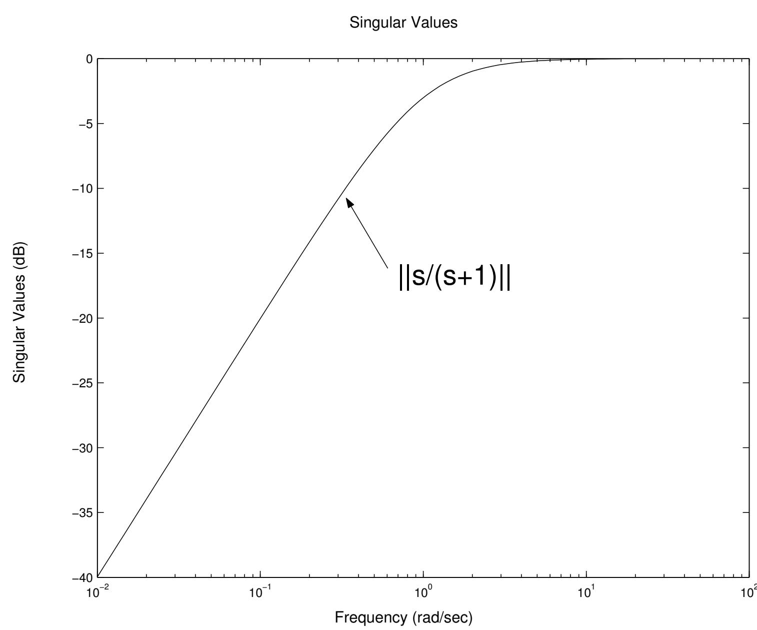

Figure 1: Template on the sensitivity function S.

- Translate this control problem using H_∞ specifications and describe it using a block-diagram representation,
- Propose a set of weighting filters from Figures 2 et 1,
- Compute the state-space representation of the corresponding $P(s)$,
- Does this standard problem meet the regularization assumptions? Give some recommendations to set-up a well-posed standard problem.
- Design the controller $K(s)$ using the Matlab Robust Control Toolbox:
  - firstly, using function `hinfsyn` (full order controller design),

- secondly, using function `hinfstruct` (fixed-structure controller design), a multi-objective formulation of the control problem and a judicious controller structure,
- compare and comment the 2 solutions.

- Redesign the control law assuming that the measurement of $\dot{y}$ is also avaialable.

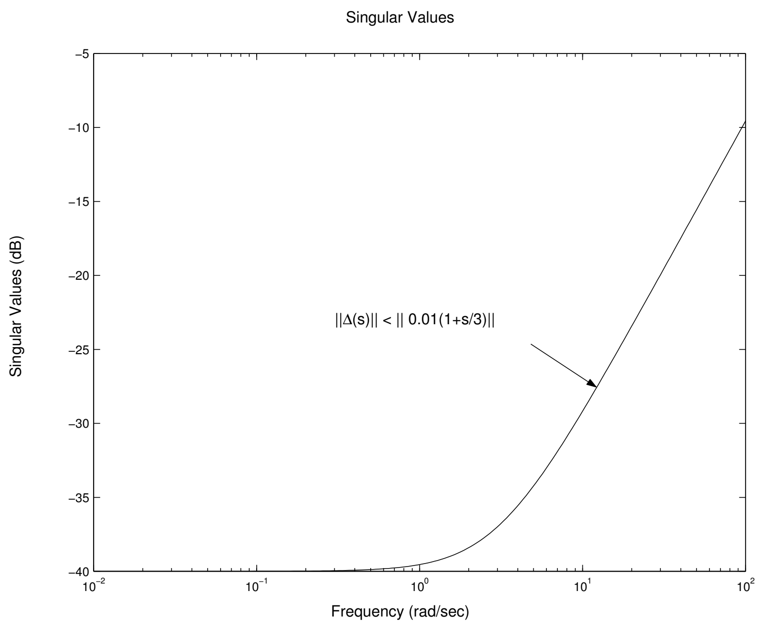

Figure 2: Uncertainty Δ upper bound.

Matlab session relative to exercise 1

```
% The plant model:
G=tf(1,[1 0 -1]);

% Standard problem without frequency weightings (minimal state-space realization)
plantnu=minreal(ss([1 -G;0 1;1 -G]));

% Weighting filters:
W1=tf([1 1],[1 0.001]);
W2=tf(0.01*[1/3 1],[1/300 1]);

% Standard problem with frequency weights:
plant=[W1 0 0;0 W2 0;0 0 1]*plantnu;

% Full order controller ynthesis:
[K,bf,gam]=hinfsyn(plant,1,1);
% gam=1.5  ==> objectives are not perfectly met (the trade-off is hard!!)

% Results analysis
S=inv(1+G*K);
W=logspace(-2,2,100);
figure
sigma(S,W);
hold on
sigma(1/W1,W);
figure
sigma(K*S,W);
hold on
sigma(1/W2,W);

% Fixed-structure desing using hinfstruct:
% A suitable structure of controller is a 3rd order with a relative degree of 1
% including one integrator (last coeff of DEN(s)=0)
kdes = ltiblock.tf('ordre3',2,3);
kdes.den.Value(end) = 0;     % set last denominator entry to zero
kdes.den.Free(end) = false;  % fix it to zero
CL=lft(plant,kdes);
pb1=CL(1,1);   % first objective
pb2=CL(2,1);   % second objective
OPT=hinfstructOptions('RandomStart',3);    % using 3 initial guess
[CL,gam]=hinfstruct(blkdiag(pb1,pb2),OPT);
% gam=1.39  ==> this 3rd order controller provides better results than
%               the 4th order (full order) controller designed on the
%               sufficient condition ||W1*S;W2*K*S||_\infty<1
K2=ss(CL.blocks.ordre3)
```

```
norm(lft(plant,K2),'inf')
% 1.77 ==> that shows the sufficient condition is not pertinent!!
```

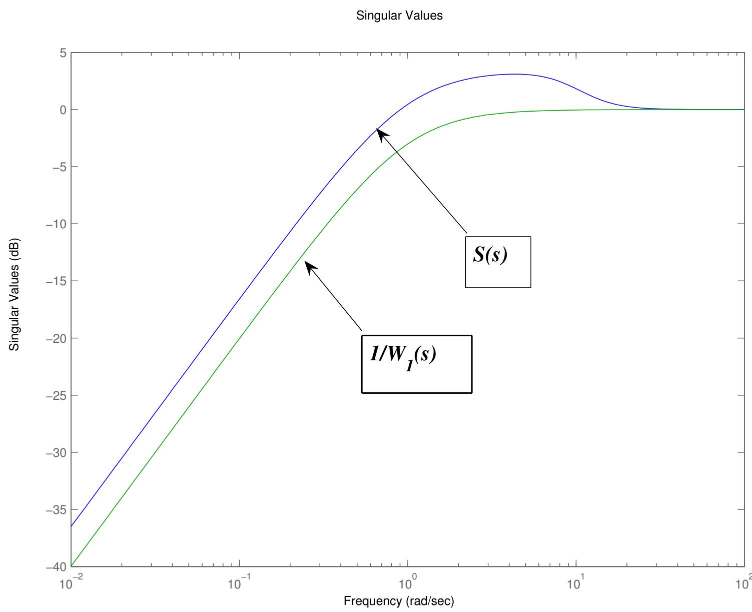

Figure 3: Result for the sensibility function S.

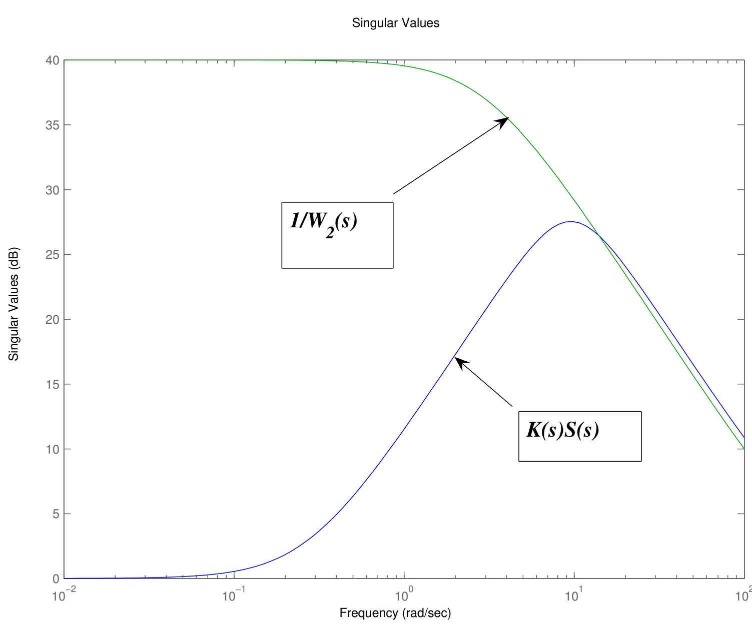

Figure 4: Result for KS.