Control of aerospace vehicles
Labwork on Matlab/Simulink - Modal, LQ and H_∞ control design

Lateral flight control design

The A/C lateral flight model G is given by the state-space represenatation:

$$\left[\begin{array}{c} \dot\beta \\ \dot p \\ \dot r \\ \dot\phi \\ \hline n_y \\ p \\ r \\ \phi \end{array}\right] = \left[\begin{array}{cccc|cc} -0.140 & 0.053 & -0.999 & 0.047 & 0 & 0.030 \\ -2.461 & -0.992 & 0.262 & 0 & 0.404 & 0.260 \\ 1.585 & -0.041 & -0.267 & 0 & 0. & -0.680 \\ 0 & 1.000 & 0.053 & 0 & 0 & 0 \\ \hline 0.0433 & -0.0003 & 0.0016 & 0 & 0.0001 & -0.0075 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 \end{array}\right] \left[\begin{array}{c} \beta \\ p \\ r \\ \phi \\ \hline dp \\ dr \end{array}\right] . \tag{1}$$

- 4 states: β (sideslip angle, rd), p (roll rate, rd/s), r (yaw rate, rd/s), ϕ (roll angle, rd),
- 2 control signals: dp (aileron angle rd) and dr (rudder angle, rd),
- 4 measurements: n_y (lateral acceleration, g), p (roll rate), R (yaw rate), ϕ (roll angle)

1 Static feedback and feedforward design

The lateral flight control law is depicted on the following Figure where `K` is static output feedback and `H` is a static (2×2) feedforward designed to met the steady state specification:

$$\lim_{t\to\infty} \left[\begin{array}{c} \beta(t) \\ \phi(t) \end{array}\right] = \left[\begin{array}{c} \beta_{consigne} \\ \phi_{consigne} \end{array}\right]$$

For each design (modal and LQ):

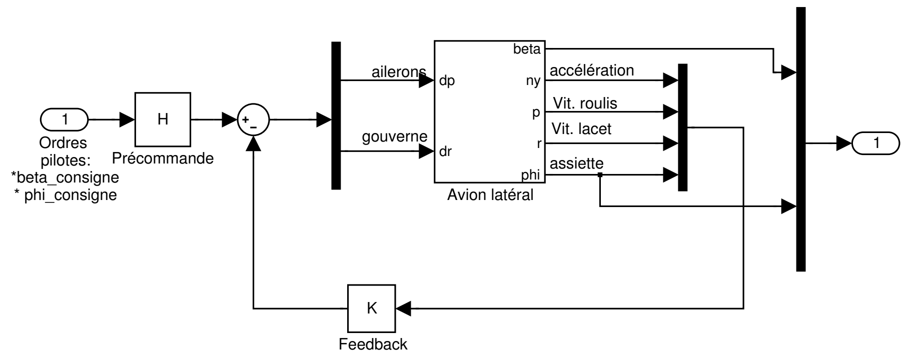

Figure 1: Lateral flight control law.

- the state feedback ($u = -K_x x$) will be first designed,
- the number of measurements (4) being equal to the number of states, the output feedback `K` equivalent to the state feedback K_x will be computed,
- then the feedforward `H` will be computed,
- finally, the step responses of the closed loop system (Figure 1) and the modulus (stability) margin will be analyzed.

1.1 Eigen-structure assignment

The requirements:

- the roll mode must be assigned to -1.1 and decoupled from β,
- the spiral mode must be assigned to -1 and decoupled from β,
- the Dutch roll mode must be assigned to $-1 \pm 1.3i$ and decoupled from ϕ.

- compute and analyze the control law assigning only the 4 eigenvalues (function `place`),

- compute and analyze the control law which meets also decoupling requirements.

1.2 LQ control

The state feedback K_x is now computed to minimize the performance index:

$$J = \int_0^\infty (q_r\, r^2 + q_\phi\, \phi^2 + dr^2 + dp^2) dt$$

- using q_r andq_ϕ (try and errror procedure), compute a state feedback K_x such that the Dutch roll damping ratio is at least 0.6 and the spiral mode is faster that 1 (rd/s). Analyze such a solution and comment decoupling properties,

Implicit referrence model: the performance index is changed to ($x = [\beta,\ p,\ r,\ \phi]^T$):

$$J = \int_0^\infty (\dot{x} - A_d x)^T (\dot{x} - A_d x) dt$$

with:

$$A_d = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -2\xi_r\omega_r & 0 & -\omega_r^2 \\ \omega_l^2 & 0 & -2\xi_l\omega_l & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ et : } \omega_r = 1.049\, rd/s,\ \omega_l = 1.64\, rd/s,\ \xi_r = 1.001,\ \xi_l = 0.61\ .$$

Justify such a perfomance index. Compute the weighting matrices Q, R and S of the equivalent LQ performance index. Compute an analyse the correspond solution.

2 H_∞ design of a 2 d.o.f control law

The H_∞ design of a 2 degrees-of-freedom control law is now considered:

$$\begin{bmatrix} dp \\ dr \end{bmatrix} = C_{\text{2ddls}}(s) \begin{bmatrix} \beta_{cons} \\ \phi_{cons} \\ n_y \\ p \\ r \\ \phi \end{bmatrix} .$$

The proposed standard problem is depicted in the following Figure. 2.

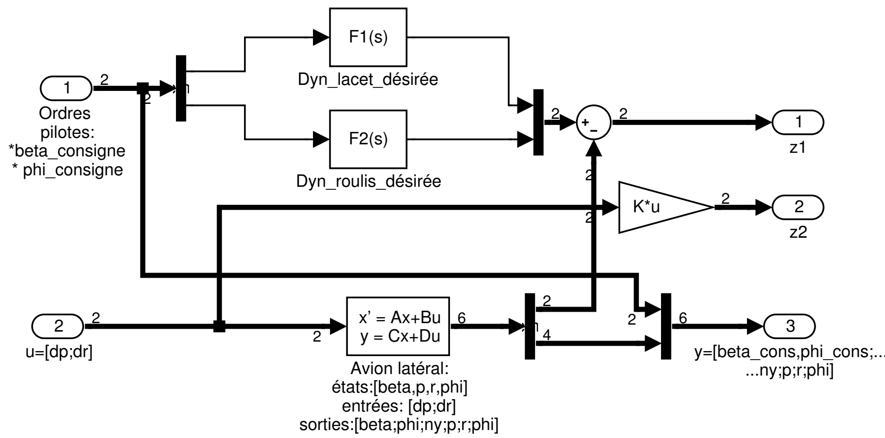

Figure 2: Standard problem for reference model following.

- comment such a standard problem,
- choose the weighting filters `F1(s)` and `F2(s)` and the gain `K` to obtain a correct closed-loop behavior (decoupling, settling times),
- design an $H\infty$ full order controller (function `hinfsyn`) and a static controller (function `hinfstruct`). Analyze both solutions: step responses to reference inputs ($\beta_{consigne}$ and $\beta_{consigne}$) and the responses to an initial condition on β (due to a lateral wind gust).