Course: Modelling and control of flexible structures
MATLAB lab-work
CAT: Telescope Attitude Control design

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Let us consider the model $M(s)$ of a space telescope around the elevation axis represented in Figure 1 (SIMULINK file `modelev.mdl`) where:

- θ is the inertial attitude of the telescope LOS (Line Of Sight),
- θ_{sat} is the inertial attitude of the holder satellite (disturbance),
- θ_m is the measurement of θ,
- $\ddot{\theta}_m$ is the measurement of the LOS inertial angular acceleration,
- u is the control signal (torque) applied on the elevation axis.

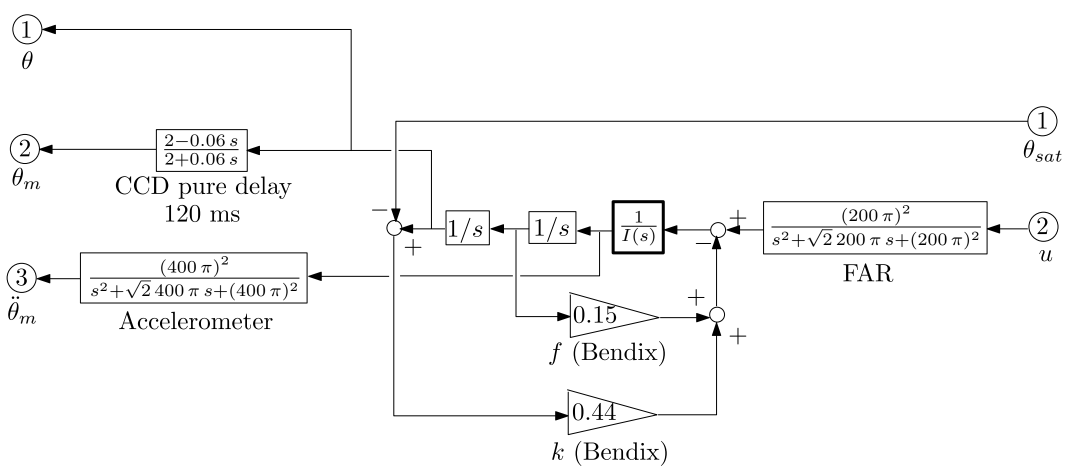

Figure 1: CAT model - elevation axis: $M(s)$.

The inverse dynamic model of the telescope $1/I(s)$ (identified model) is defined by the variable `elev` in the file `data.mat`.

The objective is to design the (simplest) control law:

$$u = K(s) \begin{bmatrix} \theta_m \\ \ddot{\theta}_m \end{bmatrix}$$

in order to dynamically isolate the LOS θ from the holder vehicule motion θ_{sat}. The performance of the control law $K(s)$, in terms of disturbance rejection, is given by $\|F_l(M(s), K(s)\|_\infty$.

1 Open-loop analysis

1.1 plot the frequency-domain response (BODE) of $1/I(s)$,

1.2 plot the open-loop disturbance rejection function (magnitude of the frequency-domain response) $\frac{\theta}{\theta_{sat}}(s) = F_l(M(s), [0 \;\; 0])$.

2 Pure performance design

Find a stabilizing controller $K(s)$ with a $-50\,dB$ performance level, that is such that:

$$\|F_l(M(s), K(s)\|_\infty < -50\,dB \;. \tag{1}$$

2.1 Using fixed-structure H_∞ synthesis (function `hinfstruct`), design a minimal order controller meeting the constraint (1).

2.2 Plot the closed-loop disturbance rejection function $F_l(M(s), K(s))$ (to be superposed to the plot of question **1.2.**).

2.3 Analyse the stability margins of the controller $K(s)$ (NICHOLS plot of the loop tranfer $L(s) = -K(s)G(s)$ and BODE plot of $1/(1 - K(s)G(s))$ where $G(s)$ is the transfer from u to $[\theta_m \; \ddot{\theta}_m]^T$ of the model $M(s)$). Conclusions.

3 Performance and robust stability design

In order to guarantee good stability margins, an additional constraint is introduced:

$$\|1/(1 - K(s)G(s))\|_\infty < 3\,dB \;. \tag{2}$$

3.1 Using fixed-structure H_∞ synthesis (function `hinfstruct`), design a minimal order controller meeting constraints (1) and (2).

3.2 Analyze the controller using the given function `catanalyse`[1].

4 Model reduction for control design

In order to save time in the design process, a reduction of the inverse dynamic model inverse $1/I(s)$ to the order 6 (i.e.: only 3 flexible modes) is required. Let us denote

[1]This function calls the nominal model $M(s)$ and a worst-case model (for a particular configuration of uncertain parameters in the sub-model $I(s)$). Type `help catanalyse` in your MATLAB command window. The objective is to obtain good stability margins on the nominal model (the NICHOLS plot of the loop tranfer $L(s)$ does not go inside the 3 dB contour) and the stability on the worst-case model.

$1/I_r(s)$ the reduced inverse dynamic model, $M_r(s)$ the augmented model obtained in substituting $1/I(s)$ by $1/I_r(s)$ in the Figure 1 (or SIMULINK file `modelev.mdl`) and $G_r(s) = M_r(s)(2:3, 2)$.

4.1 Balanced reduction: Compute $1/I_r(s)$ by reduction (truncature or matching DC gain) in the balanced realization of $1/I(s)$. Compare frequency-domain responses of $1/I(s)$ and $1/I_r(s)$. Design the minimal order controller meeting constraints (1) and (2) obtained in substituting $M(s)$ by $M_r(s)$ and $G(s)$ by $G_r(s)$. Analyze the controller using the function `catanalyse`. Conclusions.

4.2 Modal reduction: in the modal realization of $1/I(s)$ select (by visual inspection) the 3 flexible modes [2] which seem to be determinent in the evaluation of the controller (by the function `catanalyse`). Re-design the controller from this reduced model.

5 High perfomance and robust design

Find now a stabilizing controller $K(s)$ with a $-70\,dB$ performance level, that is such that:

$$\|F_l(M(s), K(s)\|_\infty < -70\,dB\ . \tag{3}$$

5.1 Design a minimal order controller meeting constraints (1) and (2). Eventually, a third constraint

$$\|1/(1 - K(s)G_p(s))\|_\infty < \infty \tag{4}$$

can be introduced to ensure the stability on the worst-case model $G_p(s)$ ($G_p(s)$ is given by `sys_val_pc(3:4,3)`, from the file `modval_cd.mat`).

[2]Function `modalred` could be used for this task.